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APPLIED MECHANICS REVIEWS

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A SURVEY OF RECENT PROGRESS IN THE THEORY OF ELASTIC SHELLS

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INTRODUCTION

THE first attempt to formulate a bending theory of thin shells from the general equations of elasticity was made by Aron in 1874, and was followed in 1888 by the successful approximate theory known as Love's first approximation. Since its inception, the formulation of the linear theory of elastic shells has repeatedly been re-examined in the literature, and has been the subject of much controversy; and even today, despite recent improvements (to which reference will be made presently), some workers in the field have expressed the view that the present status of the general theory of shells still lacks the mathematical rigor enjoyed by other special theories of elasticity, such as the theory of bending of plates.

It is the aim of this survey to present the recent progress in the theory, as well as some applications which employ novel methods of solution and/or are contributive to the development of the theory; and, in the main, only those references relevant to this purpose will be cited.[1] More specifically, this article is concerned with the equilibrium of thin elastic shells, with special emphasis placed on the linear theory; when appropriate, vibration problems are also discussed. The stability problems of shells, however, are thought to be beyond the scope of this brief survey.[2]

I. LINEAR THEORY

1. *General Theory.* For the sake of clarity, we begin by describing briefly the character of the classical theory of Love's first approximation (1). In the interest of simplicity, let a point of the middle surface of the shell be referred to by the coordinate curves ξ_1 and ξ_2 which, as lines of curvature, together with ζ (measured along the outward normal to the middle surface), form a triply orthogonal coordinate system. Further, let the displacements of a point in shell space be assumed in the form

$$U_1 = u_1(\xi_1, \xi_2) + \zeta\,\beta_1(\xi_1, \xi_2), \quad U_2 = u_2(\xi_1, \xi_2) + \zeta\,\beta_2(\xi_1, \xi_2)$$
$$U_\zeta = w(\xi_1, \xi_2) \qquad [1]$$

and recall that the stress resultants (membrane forces), the stress couples (bending moments), and the transverse shear stress resultants are defined in the conventional manner by expressions of the type

$$\begin{Bmatrix} N_{11} \\ M_{11} \\ V_1 \end{Bmatrix} = \int_{-h/2}^{+h/2} \begin{Bmatrix} \sigma_{11} \\ \zeta\sigma_{11} \\ \sigma_{1\zeta} \end{Bmatrix} \left(1 + \frac{\zeta}{R_2}\right) d\zeta\,, \qquad [2]$$

$$\begin{Bmatrix} N_{21} \\ M_{21} \end{Bmatrix} = \int_{-h/2}^{+h/2} \sigma_{12} \begin{Bmatrix} 1 \\ \zeta \end{Bmatrix} \left(1 + \frac{\zeta}{R_1}\right) d\zeta$$

where h denotes the thickness of the shell; σ_{11}, σ_{12}, and $\sigma_{1\zeta}$ refer to the appropriate components of stress; and R_1 and R_2 are the principal radii of curvature of the middle surface. The basic assumptions of the classical theory, consistent with the form of the displacements in [1], are: (i) The thickness h of the shell is small compared with the least radius of curvature $R_{\min}$ of the middle surface, i.e., $h/R_{\min} \ll 1$; (ii) the strains and displacements are sufficiently small, so that the quantities of the second- and higher-order magnitudes in the strain-displacement relations may be neglected in comparison with the first-order terms; (iii) the component of stress normal to the middle surface is small compared with other normal components of stress and may be neglected in the stress-strain relations; and (iv) the normals to the undeformed middle surface remain normal to the deformed middle surface and suffer no extension.

The first assumption, for all practical purposes, defines what is meant by a "thin" shell. The second assumption, which is inherent in the classical linear theory of elasticity, ensures the linearity of the resulting differential equations. The third and fourth assumptions, respectively, imply the neglect of transverse normal stress and transverse shear deformation. It is the latter assumption which enables one to relate β_1 and β_2 to the middle surface displacements u_1 and u_2 and partial derivatives of w. The theory of Love's first approximation is based on the four assumptions mentioned above, with a further stipulation that the ratio of ζ/R is neglected in comparison with unity in the expressions of both stress resultants and the strain-displacement relations.

A very general and consistent derivation of Love's first approximation (which is also valid for nonorthogonal coordinate curves on the middle surface) is contained in a recent treatise by Green and Zerna (2, Chap. 10), where extensive use is made of tensor calculus. An exposition of the subject may also be found in (3) where, in addition to a derivation of

[1] A comprehensive bibliography on the subject has recently been compiled by Nash (46). Numbers in parentheses indicate references at the end of the paper.

[2] Some aspects of the stability problems of shells have been considered in a recent article by L. H. Donnell, AMR 5, 289–290, 1952.

Love's first approximation, a brief survey is included on the theories of Love's second approximation, Flügge-Byrne, Trefftz, and Synge and Chien. Mention should here be made of a recent Russian book on the subject by Goldenveizer (4) which has not, as yet, been available to the writer.

While the third assumption stated above may be considered reasonable, the third and fourth assumptions taken together are not entirely satisfactory, and, as already established in the cases of beams and plates, the effect of the latter may be of considerable importance in some classes of problems.

With a view toward improving Love's first approximation, some investigators have sought new derivations of the classical theory by retaining in the stress resultants and the strain-displacement relations, terms of the order $(\zeta/R)^2$ as compared to unity (5,6). Others, by discarding assumptions (iii) and (iv), have formulated *suitable* stress-strain relations, thus broadening the scope of the theory by including the effects of transverse shear deformation, rotatory inertia, and normal stress (7,8,9). This latter may be accomplished by including additional terms involving ζ and ζ^2 in the expression for U_ζ in [1], in which case β_1 and β_2 (since the transverse shear strains do not vanish) represent the changes of slope of the normal to the middle surface. As has been pointed out in (8,9), in the event that a theory is sought in which only the effect of transverse shear deformation (and rotatory inertia) is retained, the form of the displacements in [1] needs no improvement. It should also be noted that the expressions for the boundary conditions (four at each edge of the shell), which are appropriate in such theories as Love's first approximation (1) and Flügge-Byrne (5,6), require modification in the improved theory of (7,8,9) where the effects of transverse shear deformation and normal stress are accounted for. As given in (9), the number of independent boundary conditions at each edge of the shell is five and the additional requirement (similar to that in the E. Reissner theory of plates) is consistent with the order of the differential equations of the improved theory.

Notice should be taken of the numerous efforts made in an attempt to arrive at an improved theory of shells by expanding the various quantities in terms of powers of the thickness coordinate. In recent times, many authors have adopted this technique and, in parts of the theory, some have retained, often inconsistently, terms to as much as second or higher orders in (ζ/R). That this viewpoint is meaningless, owing to the approximations made in the rest of the theory, and does not lead to any definite improvement is evident from the results of these works: (10,11). In this connection and with regard to other inconsistent approximations, the comments of E. Reissner and Truesdell (12) are particularly enlightening.

Two other aspects related to the general theory are worth mention: One is the use of appropriate expressions for the strain energy of a thin shell for the purpose of obtaining approximate solutions. Here again, expressions are often employed which are not entirely consistent. While the expression for the strain energy within the scope of Love's first approximation is well-known, a number of recent authors have sought to obtain, either fully or in part, an improvement for the expression of the strain energy which is already contained in Byrne's work. In dealing with continuous shells, such as continuous cylindrical panels, we again refer to Byrne who, with considerable generality, obtained all the geometrical requirements for such shells.

2. *Shells of Revolution.* The differential equations of the theory in the form given by Love (1) were not, even in special cases, amenable to analytical treatment. In 1912, H. Reissner (13), by a judicious choice of the dependent variables, expressed the deformation of a spherical shell under axisymmetric loading in terms of two ordinary second-order differential equations and indicated their solution by means of series involving exponential functions. In the following year, Meissner (14) noted that these equations of H. Reissner were of the hypergeometric type and could be generalized to include all shells of revolution generated by curves of constant radii of curvature. In a subsequent extension, Meissner (15) showed that a similar reduction of the general fourth-order differential equation to two second-order equations was possible for any shell of revolution, provided the thickness of the shell varied in a prescribed manner (Meissner's condition). This latter paper also contains exact solutions for toroidal and spherical shells, and for the conical shell with linearly varying thickness. In (13), H. Reissner, in addition to observing that the nonhomogeneous solution of the differential equations (due to the presence of surface load) may be approximated by a corresponding membrane solution, suggested (as noted above) the possibility of an asymptotic solution of the equations in powers of the ratio of shell thickness to a representative dimension of the shell. The integration of the H. Reissner-Meissner differential equations for the general shell of revolution, by classical methods of asymptotic integration, has been given by Hildebrand (16). Such solutions, although very useful in many cases, are not valid whenever the region of interest includes a point (such as the apex of spherical shells) at which a singularity occurs in the differential equations.

The derivation of the axisymmetric deformation of shells of revolution with small displacements and the resulting differential equations were reconsidered in 1949 by E. Reissner (17) which, although differing only slightly from those of H. Reissner-Meissner, offer certain advantages not revealed in earlier formulations. Subsequently, the differential equations of the linear theory in the form given in (17) were combined into a single complex differential equation which is valid if the "Meissner condition" holds (18). In this form, the differential equations of shells of revolution are readily amenable to treatment by a method of asymptotic integration due to Langer (19,20), and yield solutions valid at singularities of the equations. Such solutions have recently been obtained for toroidal (21) and ellipsoidal (22) shells.

The inclusion of the effect of transverse shear deformation in the axisymmetric theory of shells of revolution leads to a system of two coupled ordinary differential equations which are considerably more complex than the H. Reissner-Meissner equations (23). As pointed out in (23), the solution of these equations may again, with the aid of Langer's work (20), be deduced which is valid at singularities of the differential equations. These results have very recently been applied by DeSilva (24) to the general paraboloidal shells of revolution of uniform thickness, the solution being valid at the apex of the shell.

3. *Shallow Shells.* Consider the parallel projection of the middle surface coordinate curves on a plane P along the perpendicular to this plane, and let z be the distance (along the perpendicular to P) between two corresponding points on the middle surface and the plane P. A shell (or a segment of a shell) is said to be shallow whenever z/l (l being a reference length) and its first partial derivatives are small in comparison with unity. In the case of a shallow spherical shell (spherical cap), for example, P is the plane of the bounding circle (base plane), l may be taken as the radius of curvature, and z is the height of the shell from P to a point on the middle surface.

As a consequence of the above approximation, the differential equations of the bending theory (such as Love's first approximation) further simplify and, in the absence of inertia forces, may be reduced to two simultaneous partial differential equations involving an Airy stress function F and the axial displacement w_z. A similar reduction also is possible in the case of the improved theory of (7,8,9). Starting with Love's first approximation, a systematic derivation of the theory of shallow shells and the resulting differential equations for F and w_z, contained also in the work of Marguerre (25), are

ven in Green and Zerna (2, Chap. 11). More recently, it has een shown by E. Reissner (26) that the reduction to two imultaneous equations for F and w_z is also possible in the ase of transverse vibration of shallow shells, when longitudi- al inertia terms, with negligible error, are omitted. Here, ong the existing specific solutions, mention should be made E. Reissner's solution for shallow spherical shells (27,28).

4. *Other Aspects of Theory.* The solution of shell prob- ms, using the bending theory, generally involves a great eal of mathematical difficulty and, except for cylindrical hells, relatively little progress has been made in recent times obtaining solutions to nonsymmetric problems of shells, as videnced by the recent papers of Hoff (29) and Berry (30), here other references may be found.

With reference to circular cylindrical shells, Donnell's quations (derived in 1933), which, in view of their relative implicity, have received increasing attention in this country, ave evidently remained somewhat unknown abroad (31). Equa- ions equivalent to Donnell's may also be deduced as a spe- ial case of the differential equations for bending of cylindri- al shells presented in (2, Chap. 13). With regard to the ac- uracy of simplified equations of the theory, some recent re- ults for axisymmetric propagation of elastic waves in cy- indrical shells are of interest: the agreement, for phase elocities of propagated waves, in all modes of motion, be- ween the predictions of the two systems of equations (which, pon the neglect of transverse shear deformation and rotatory ertia, reduce to those of Love's first approximation and onnell's) employed in (32), as well as the available nu- erical results of the three-dimensional solution of the prob- m (33), is remarkable; in (32), the displacements, as pre- icted by the two systems of equations, were also found to be n good agreement. The accuracy of Donnell's equations (as ompared to Flügge's) has recently been discussed by Hoff 34), and, in connection with the problem treated in (35), it as further been shown by Cooper (36) that a refinement of onnell's equations (by including the effect of transverse hear deformation, as in (32)), which may be significant for the ndial displacement, has no effect on the stresses.

An account of the linear membrane theory of shells (which as been adequately explored) in general curvilinear coordi- ates is given in (2, Chaps. 12, 14), and this aspect of the heory has also been discussed in a recent article by Flügge 37). Reference may also be made to a recent paper by E. eissner (38) which contains an exposition of the membrane heory and its application to a number of examples of interest n architectural design.

II. NONLINEAR THEORY

While a great deal of work has continuously been done in he linear theory of thin shells, surprisingly little information is available about shells with finite displacements. Among the various contributions which deserve special attention are the papers of Synge and Chien (39), and Chien (40). The latter, which has been criticized by Green and Zerna (7) and Goldenweiser and Lourye (41), has recently been re-examined by Reiss (42) who evidently has discovered several errors in Chien's work. As remarked in (7), Chien's theory, being developed from the "intrinsic" point of view, does not make use of the displacement vector and hence may be handicapped in practical applications where the boundary conditions are often expressed in terms of displacements. Thus, starting from the general (microscopic) equations of elasticity and employing the same assumptions as Synge and Chien, Reiss (42, part V) has presented a formulation of finite deformation of thin shells in terms of displacements: First, the microscopic differential equations of equilibrium in terms of displacements are deduced and then, by regarding this system of three-dimensional equations as defining an initial value problem (the middle surface being the initial manifold) and expanding the displacements in a power series of the thickness coordinate, the problem is reduced to a two-dimensional one. In this manner, the problem is thus characterized by a system of three second-order elliptic partial differential equations (occupying more than three pages) which, together with the values of the displacements and their first normal derivatives at the middle surface (if known), determine the displacements at every point in the shell space. Although the approach adopted in (42) represents some progress, it is believed that a suitable formulation of the general elastic shell as a three-dimensional problem (even for small displacements), which would lead to definite conclusions with regard to the validity of the bases of the various approximate theories of shells, is yet to be achieved.

For shells of revolution with finite displacements, an axisymmetric theory, based on assumptions (i), (iii), and (iv) stated earlier, has been formulated by E. Reissner (43), where reference to previous work on the subject may be found. In (43), two simultaneous second-order differential equations, which by linearization reduce to the H. Reissner-Meissner equation in the form given in (17), are deduced and special attention is devoted to the problems of circular cylindrical and spherical shells. In addition, criteria for the range of applicability of the linear theory are presented with the conclusion that the linear theory has a wider range than customarily thought. This latter is also considered in (44) in connection with a problem of toroidal shell.

Finally, mention should be made of a system of differential equations of shallow shells, due to Marguerre (25), which includes nonlinear effects and which has recently been utilized in (38) and (45).

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(See also Revs. 2814, 2824, 2825, 2828, 2830, 2836, 2845, 2854, 2859, 2913, 2917, 2919, 2951, 2954, 2955, 2985, 2993, 3016, 3017, 3065, 3066, 3067, 3068, 3069, 3132)

Book—2796. Kopal, Z., Numerical analysis, New York, John Wiley & Sons, Inc., 1955, xiv + 556 pp. $12.

This is an excellent book and a valuable addition to the analyst's shelf. Emphasis is on the application of numerical techniques for approximation by polynomials to functions of a single variable. There is no attempt to treat the subject of matrixes, solution of linear and nonlinear algebraic equations, and multivariable problems.

The introduction (chap. I) is a short history and a philosophy of numerical analysis. Its study will pay rich dividends to all workers, both pure and applied. The author's aim is to present various methods for the solution of problems. He rightly points out that there is no panacea, and the question of a best or preferred formula has little meaning except in reference to a specific problem. Numerical analysis is not a collection of formulas. It is a scientific discipline for the development of processes for the solution of specific problems. The author's objective is to inculcate this discipline, and this he does admirably well.

At the end of each chapter there is a valuable collection of bibliographical and historical notes and a list of problems. Some of the problems are direct applications of the techniques presented. Others are designed to probe more deeply into the subject, and, in addition, there are some for which the answers are not known. Development of these topics could lead to research theses.

Chapters II, III, and IV deal with polynomial interpolation, differentiation, elementary methods of integration, and numerical solution of ordinary differential equations. The subject of mechanical quadrature (Gaussian, Chebyshev, etc.) is thoroughly treated in chap. VII.

Chapters V and VI treat the subject of boundary-value problems, while numerical solution of integral and integro-differential equations is presented in chap. VIII. The latter three chapters cover important material, and this is the first time topics of this type are to be found in the English language. Inclusion of a chapter on matrixes would have considerably enhanced the volume, as this would provide a better basis for the discussion of these problems. However, a short bibliography is given in Appendix V.

Further material is presented in the Appendix. Appendix I develops finite difference formulas using the operational approach. Appendix II is an introduction to optimum interpolation and economic representation. There is a serious omission in the discussion of Chebyshev polynomials as a second work of C. Lanczos ["Tables of Chebyshev polynomials $S_n(x)$ and $C_n(x)$", NBS, AMS 9, 1952] is not mentioned. Appendixes III and IV give coefficients of the Lagrangian formulas for numerical differentiation and coefficients for various types of mechanical quadrature formulas.

Y. L. Luke, USA

2797. Duncan, W. J., Reciprocation of triply-partitioned matrices, *J. roy. aero. Soc.* **60**, 542, 131–132 (Technical Notes), Feb. 1956.

In this brief note author gives a method for finding the reciprocal of a matrix which is triply partitioned horizontally and vertically in such a manner that the submatrixes in the principal diagonal are square, but these matrixes need not be of the same order. A preliminary rearrangement of the matrix may be helpful.

From author's summary by M. Maletz, USA

2798. Guest, J., The solution of high order polynomial equations by matrix iteration, *Aero. Res. Labs., Melbourne Austral. Rep.* SM. 236, 8 pp., Dec. 1955.

Author extends Aitken's method and develops iteration methods to determine successively the roots of a polynomial equation. Method should be of use for work on high-speed digital computers. Author states that roots not covered in his paper may occur (only rarely) and that, in that case, iteration methods will not be suitable.

D. ter Haar, England

2799. Honnell, P. M., and Horn, R. E., Matrices in analogue mathematical machines, *J. Franklin Inst.* **260,** 3, 193–207, Sept. 1955.

A system of n-th order differential equations with constant coefficients are transformed by a suitable matrix operator to a set of first-order differential equations. Each element of the resultant matrix is then represented by a triod electron tube so that the resultant analog structure is in one-to-one correspondence with the asymmetrical matrix structure. Several forms of these matrixes may be established in a search for the electrically most stable analog representation.

G. Kron, USA

2800. Anonymous, Computing mechanisms. Part I, Part II, *Prod. Engng.* **27,** 3, 200–201, Mar. 1956; 4, 180–181, Apr. 1956.

Analog computing mechanisms are capable of virtually instantaneous response to minute variations in input. Basic units, similar to the types shown, are combined to form the final computer. These mechanisms add, subtract, resolve vectors, or solve special or trigonometric functions. Other mechanisms for multiplying, dividing, differentiating or integrating are presented in part II.

From summary

2801. Gotlieb, C. C., Free use of the Toronto computer, and the remote programming of it. I, *Computers and Automation* **5,** 5, 20–25, May 1956.

2802. Grensted, P. E. W., Stability criteria for linear equations with time-varying coefficients, *J. roy. aero. Soc.* **60,** 543, 205–208, Mar. 1956.

Author presents possible but not rigorous approach.

B. Smilg, USA

2803. Moon, P., and Spencer, D. E., On the classification of the ordinary differential equations of field theory, *Quart. appl. Math.* **14,** 1, 1–10, Apr. 1956.

The coordinate systems in which Laplace and Helmholtz equations are separable are tabulated. Paper provides the separation equations for the eleven simply-separable systems of Eisenhart, the eleven symmetric cyclide systems, and eighteen cylindrical systems. All separation equations for these coordinate systems reduce to nineteen distinct equations of the Bocher type, which are presented together with their solutions.

C. R. Mischke, USA

2804. Rutishauser, H., On the instability of methods for the integration of ordinary differential equations, *NACA TM* 1403, 15 pp., Apr. 1956. (Translation of *ZAMP* **3,** 65–74, 1952.)

Some examples are given in which a differential equation is replaced by a difference equation of higher order, giving rise to extraneous solutions which grow faster than any solution of the given equation; due to errors, the numerical solution follows the extraneous solutions (instability). A general criterion for stability is derived and illustrated.

H. D. Block, USA

2805. Price, P. J., Perturbation theory for the one-dimensional wave equation, *Proc. phys. Soc. Lond.* (*A*) **67,** 412, 383–385, Apr. 1954.

Author presents a method for obtaining the ground states in an eigenvalue problem to any required order in terms of the unperturbed function. One sets

$$\psi(x) = \psi_0(x)\exp\left[\int_a^x \Omega(x')dx'\right]$$

and substitutes in the eigenvalue equation. The solution is valid if the perturbing potential does not alter the character of the solution. The method is demonstrated for the calculation of the phase shifts of waves scattered by a central potential.

E. Koenigsberg, England

Book—2806. Petiau, G., The theory of Bessel functions with respect to their applications to mathematical physics [La theorie des fonctions de Bessel exposee en vue de ses applications a la physique mathematique], Paris, Centre National de la Recherche Scientifique, 1955, 477 pp. 2500 francs.

In this book the author has collected much of the information available on Bessel functions, with particular regard to those properties of Bessel functions of interest to applied mathematicians. Much, but not all, of the recent literature is included in this monograph, and there are valuable references to numerical tables. The presentation ranges from detailed proofs of the principal properties to brief references to the literature in the case of less important items.

Contents: Bessel coefficients, Bessel functions of integral order. Bessel functions of order zero. Bessel functions of arbitrary order, series representations. Bessel functions of arbitrary order, integral representations. Asymptotic expansions of Bessel functions. Addition and multiplication theorems. Zeros of Bessel functions. Integrals involving Bessel functions. Neumann polynomials and series, Lommel functions, Kapteyn series. Functions related to Bessel functions. Bessel integral functions. Kelvin functions. Fourier-Bessel series and integrals. Schlömilch series. Orthogonal polynomials and Bessel functions. Differential equations whose solutions can be expressed in terms of Bessel functions. Kepler's problem, Bessel functions of several variables. Bernoulli's problem. Applications of Bessel functions to random-walk problems. Heat-conduction problems. Vibration of a circular membrane, and of an elastic plate, finite Hankel transforms. Applications of Bessel functions to diffraction problems. Applications to electromagnetic wave propagation. Applications to wave mechanics. Numerical tables. Graphs.

Courtesy of Mathematical Reviews A. Erdelyi, USA

2807. Richter, W., Transformations of coordinates with the help of a vanishing line nomogram and its application for solving certain differential equations by graphical methods (in German), *Ost. Ing.-Arch.* **8,** 1, 39–47, 1954.

2808. Volkov, E. A., One way of increasing the accuracy of the method of nets (in Russian), *Dokladi Akad. Nauk SSSR (N.S.)* **96,** 4, 685–688, June 1954.

Book—2809. Rand Corporation, A million random digits with 100,000 normal deviates, Glencoe, Ill., The Free Press, 1955, xxv + 36 pp. $10.

These tables published by Rand Corporation deserve to be mentioned because today there is an increasing need for adequate mathematical tools to help solve problems by experimental probability procedure. The so-called Monte Carlo methods are among those which can make good use of tables on random numbers. Distinguishing features of these tables are their size and their accuracy. The tables were constructed primarily for use with punched card machines.

M. Castellani, USA

2810. Koenigsberg, E., Queuing with special service, *Operat. Res.* **4,** 2, 213–220, Apr. 1956.

Special service queuing involves two types of arrivals: the

normals can be serviced at any counter, while the specials can be serviced at only m out of a counters. Assuming no priorities and a ratio r of special arrivals, author discusses relations between probabilistic states to be used in finding optimum value of m. Charts of mean waiting time versus m are shown, with and without restriction of normal arrivals to non-special counters. A Monte Carlo solution of an actual problem of loading petroleum-products trucks, some for new products requiring special loading equipment, is indicated for $a = 14$, $m = 1$, $r = 0.1$ Generalizations are made to several types of special service. Author's references to previous operations research papers by Cobham, Morse, *et al.* and earlier *Bell Syst. tech. J.* articles by Riordan and Molina are recommended for fuller understanding of queuing theory.

E. C. Varnum, USA

2811. Eisemann, K., Linear programming, *Quart. appl. Math.* **13,** 3, 209–232, Oct. 1955.

This article is an excellent exposition of the general theory and methodology of linear programming. The presentation is very lucid for one familiar with the mathematics of matrixes. The coverage of special conditions and underlying theorems is very complete but there are no actual concrete examples given except those of a rather abstract nature. Some mention is made of the use of electronic computing for linear programming although, again, no specific examples are given.

Article is fine for those wishing a basic mathematical understanding of the tool, linear programming (which is a special case of the old "method of steepest descent"), but it is a poor article for the engineer who is a little rusty in matrix algebra and wishes to determine from a practical viewpoint what linear programming can do for him.

J. H. Davidson, USA

Book—2812. Chalmette, M., English-French vocabulary of aeronautical terms (Aircraft Engineering Monographs), London, Aircraft Engineering, Bunhill Publications Ltd., 36 pp. 4 s.

See AMR 4, Rev. 4058.

Mechanics (Dynamics, Statics, Kinematics)

(See also Revs. 2905, 2945, 3034, 3142)

Book—2813. Hartman, J. B., Dynamics of machinery, New York, McGraw-Hill Book Co., Inc., 1956, xvi + 283 pp., $7.50.

This text is a recent addition to the McGraw-Hill Series in Mechanical Engineering, joining Beggs', Den Hartog's, and Schlichting's books. Prepared for advanced undergraduates and graduate students, it treats the subject matter with careful competency without either insulting the theoretically Inclined readers' intelligence with irrelevant elementary details or wandering off to sophisticated mathematics of little use to the practical man and designer. Students receive considerable useful guidance and detailed instructions which, however, give a somewhat dogmatic impression to the practicing engineer. Author emphasizes the importance of the development of a professional approach and gives many examples of "engineering logic".

The two introductory chapters review basic principles of dynamics in a concise and commendable manner. These are followed by discussions of inertia forces and of mechanical vibrations on a mostly undergraduate level. Extensive treatment of balancing, transients, and control systems comprise the last chapters.

Effective use of vector arithmetic, Fourier series and Laplace transforms, no discussion of nonlinear vibrations, and omission of the Lagrangean formulation are noted. The book is provided with a wealth of examples, to many of which answers are given.

V. G. Szebehely, USA

Book—2814. Rubinowicz, W., and Krolikowski, W., Theoretical mechanics [Mechanika Teoretyczna], Warszawa, Państwowe Wydawnictwo Naukowe, 1955, 422 pp. zł 26.

Thorough presentation of basic principles of theoretical mechanics in four chapters refers to following subjects: I. Kinematics and dynamics of a material point (velocity, acceleration, Galileo's principle, Newton's equations and their applications in relativistic mechanics and various types of vibration, Lorentz' force, principles of conservation of energy, potentials): II. Mechanics of a system of material points (equilibrium of forces, applications of Newton's equations, d'Alembert's principle, virtual work, Lagrange's equations and their applications, principles of Hamilton, Maupertuis and Jacobi): III. Hamiltonian equations and their variational transformations (their interpretation by Poisson and Jacobi, examples of Hamilton-Jacobi equations in relativistic mechanics); IV. Mechanics of rigid bodies (velocity and acceleration, impact and kinetic energy, tensors of second degree, equations of motion, equilibrium of forces, Euler's equations, examples of motion around a point, applications in geomechanics).

Full-page pictures of Newton, d'Alembert, Lagrange, and Hamilton are inserted.

J. J. Polivka, USA

Book—2815. Selényi, P., and Jánossy, L., (editors), Collected works of Roland Eötvös [Roland Eötvös Gesammelte Arbeiten], Budapest, Akadémiai Kiadó, 1953, lxxx + 384 pp.

This memorial volume contains 10 German and 1 French paper written by R. Eötvös, one of the greatest Hungarian physicists.

The well-known equivalence principle of the general theory of relativity (i.e., an accelerated frame of reference is equivalent with a gravitational field) uses the equivalence of the gravitational (m') and inertial (m) mass. Eötvös' experiments described in this volume showed that the difference between m and m' referred to m' is less than half millionth of one per cent. His method was based on the determination of the direction of the force acting on the body (its weight vector), since m/m' is proportional to the angle between the resultant force and the gravity force. Carefully performed experiments with torsional pendulums are described in great detail. Methods and results are also given in connection with the experimental determination of the variation of weight in the horizontal plane. Geological research performed in Hungary and related to the mass distribution under the surface of the earth illustrates applications.

The volume is a testimonial of the thoroughness, exactness, and classical preciseness of a great theoretical physicist, showing infinite patience for experimental details. The titles of the papers included in the volume are: Surface tension and intermolecular forces. The attraction of the earth on various materials. Measurements of long periods. Investigations of gravity and earth magnetism. Potential surfaces and the variation of the gravity and of the magnetic force. Determination of the gradient of the weight vector and of its potential surfaces with torsional balance. The potential surfaces and the gradients of the weight vector on the Lake Balaton. Report on geodetic work in Hungary. Research with the torsional balance. Experimental results of measurements of forces acting on moving bodies. The proportionality between inertia and gravity.

V. G. Szebehely, USA

2816. Györgyi, G., Theory of vibration of torsional balance of Eötvös-Selényi (in German), *Acta Phys. Hung. Budapest* **4,** 1, 79–86, 1954.

2817. Freudenstein, F., On the maximum and minimum velocities and the accelerations in four-link mechanisms, *Trans. ASME* **78,** 4, 779–787. May 1956.

Given a four-bar linkage $ABCD$; DA is the fixed, AB the driving, BC the connecting, CD the follower link, the pole P is the intersection of AB and CD, O that of BC and AD, PO is the colline-

ation axis. Author is interested in the ratio of the angular velocity of *AB* and *CD* and proves by geometrical reasoning that at an extreme value *OP* is perpendicular to *BC*. Linkages are constructed with prescribed extreme value. Extension is made to acceleration analysis of the linkage. Application to slider-crank mechanisms is shown. Appendix contains proof of the main theorem by means of Bobillier. O. Bottema, Holland

2818. Dahm, B., Coordinate transformer for rotational velocities (in German), *ZVDI* **98,** 9, 361–366, Mar. 1956.

Author presents a new mechanical coordinate transformer which consists essentially of a special spherical mechanism with three rub rollers and two rudder rollers, rigidly coupled between themselves but permiting inclination of the sphere axis. The instrument is driven by a spring-mounted electrometer (2 kw). The mechanism with a special conical differential may be utilized for quickly changing the direction of the rotation or as a *sin-cos* transformer. The combined *sin-cos* transformer is also shown.

Because of the high natural frequency and the very short eigentime, the same instrument may also be utilized in regulating practice or as a central mechanism.

The explanation is illustrated by several designs.

D. Rašković, Yugoslavia

2819. Tamamshev, A. A., Problem of the geometric synthesis of five member mechanisms having members which change dimensions (in Russian), *Trudi Inst. Mashinoved.* **14,** 56, 20–34, 1955.

2820. Rappaport, S., How direction of rotation affects cam profiles, *Mach. Design* **28,** 7, p. 120, Apr. 1956.

2821. Waclawek, M. J., General equation for solving all planetary gear ratios, *Prod. Engng.* (Product Design Handbook for 1956), 26, 11, E12a, Oct. 1955.

2822. Heilig, R., Statics of heavy ropes (in German), *Stahlbau* 23, 11, 253–258, Nov. 1954; 12, 283–291, Dec. 1954.

2823. Giles, C. G., and Lander, T. W., The skid-resisting properties of wet surfaces at high speeds: Exploratory measurements with a small braking force trailer, *J. roy. aero. Soc.* **60,** 542, 83–94, Feb. 1956.

2824. Kroeger, R. O., Graphical method for shoe brake design, *Prod. Engng.* **27,** 3, 142–147, Mar. 1956.

Center-of-pressure concept determines the equilibrium of forces quickly and accurately. Included are six sample problems of fixed-anchor and floating-anchor internal shoe brakes, design of a two-shoe brake in which one shoe energizes the other, and graphs for simplifying calculations.

From author's summary

2825. Spotts, M. F., Long-shoe block brake with floating anchor, *Trans. ASME* **78,** 4, 789–794, May 1956.

A force analysis is presented assuming pressure pattern symmetrical to lining arc to find shoe tip force. Assumption simplifies problem but depends on coefficient of friction remaining constant. Force polygons are used to achieve desired design but do not permit further analysis for effect of wear and friction.

C. R. Freberg, USA

2826. Vrublevskaya, I. N., Certain criteria for the equivalence of trajectories and semitrajectories of dynamic systems (in Russian), *Dokladi Akad. Nauk SSSR (N.S.)* **97,** 2, 197–200, July 1954.

2827. Fung, Y. C., The analysis of dynamic stresses in aircraft structures during landing as nonstationary random processes, *J. appl. Mech.* **22,** 4, 449–457, Dec. 1955.

A statistical analysis based upon the principles of generalized harmonic analysis is proposed that regards the landing gear impact load as a nonstationary process. The major objective of the paper is to provide a statistical analysis applicable to cases in which simplified landing analyses cannot be justified. The ensemble means and the correlation functions of landing impacts are defined and their experimental determination from flight or drop tests is discussed. From these, the mean stress and the root-mean-square deviation from the mean are computed. The results are used to find the most probable maximum stress attained in a given number of landings, or the most probable total number of landings a given aircraft can withstand. A stress envelope can be derived which represents the distribution of the severest stress in the structure for a large number of landings.

Paper represents a statistical approach to the landing problem analogous to that previously applied to the gust problem, and suggests the possibility of applying a similar procedure to other applied loads. R. L. Bisplinghoff, USA

2828. Lin, H., On the convergence of an iterative procedure in solving dynamic response problems, *J. aero. Sci.* **23,** 4, 391–393 (Readers' Forum), Apr. 1956.

Author shows the rate of convergence of an iterative procedure in the solution of a transient motion of a single spring-mass system with damping. T. H. Lin, USA

2829. Reznikovskii M. M., and Vostroknutov, E. G., Dynamics of rolling of a rubber ring on a rigid support (in Russian), *Zh. tekh. Fiz.* **24,** 6, 997–1007, June 1954.

Authors consider previous contributions by Ishlinsky who investigated rolling of rigid roller on imperfectly elastic supports and obtained formulas for cases of elastico-viscous support (Maxwell's body) and for visco-elastic support (Kelvin's body). Authors give experimental results and investigate the conditions of applicability of the Ishlinsky's formulas. Paper includes detailed description of the apparatus and the techniques.

Obtained experimentally are: Resistance moment to rolling; radial deflection of a rubber layer versus radial pressure on the ring; rolling velocities and the temperatures. Experiments were divided into two series. The first one consisted of comparing mechanical losses of rolling rings made of rubber with approximately the same friction coefficient with reference to metal, though hysteresis properties were different. The second set pertained also to mechanical losses. The rings were made of rubber with assigned hysteresis values. Coefficients of friction were different. Authors' observations were: When radial load on the ring is kept constant then the mechanical losses per unit of volume are proportional to the inner friction of rubber and inversely proportional to its dynamic modulus. If the work, per unit of volume, due to cycling loading is kept constant, then the mechanical losses per unit of volume are proportional to the inner friction of rubber and inversely proportional to its dynamic modulus. Hence the condition of constant radial load is equivalent to the condition of constant assigned work due to cycling loading under uniform compression. V. A. Valey, USA

Servomechanisms, Governors, Gyroscopics

(See also Revs. 2828, 3015, 3016, 3017)

2830. Poggi, L., Fundamentals of the theory of servomechanism (in Italian), *Pubbl. Fac. Ing. Univ. Pisa* (16) no. 753, 249 pp., 1955.

A very comprehensive introduction to the theory of control mechanisms, presented for engineers and arranged under the following chapter headings: (1) Fundamentals; (2) The method of Laplace transforms; (3) The transfer function; (4) The transfer locus and its interpretation; (5) Plotting of transfer loci; (6) Examples of transfer functions for elementary mechanisms; (7) Errors

and their reduction; (8) Transitors; (9) Hints for the designer.

Presentation is elementary and extremely clear; theory is illustrated by many examples. Printer's contribution is inadequate: study of the paper is rendered difficult by numerous misprints and by a queer way of numbering figures (of the 56 figures, 28 are denoted by Fig. 1). H. Ziegler, Switzerland

2831. Yates, E. C., Jr., Theoretical analysis of linked leading-edge and trailing-edge flap-type controls at supersonic speeds, *NACA TN* 3617, 40 pp., Mar. 1956.

Linear supersonic wing theory is applied to the calculation of the hinge moment of linked leading-edge and trailing-edge controls. The calculations carried out for a number of cases indicate that the hinge moments can be considerably reduced by allowing the moment on the leading-edge control to counteract that on the trailing-edge flap. The reduction is not greatly affected by Mach number variation for those considered (M = 1.41 and 1.96). Author allows in some cases for the effect of control thickness. Recent work by Sweeney [*M. I. T. naval supersonic Lab. Rep.* TR 109] indicates that affects of shock-boundary-layer interaction will also significantly influence the design of a minimum hinge-moment linked-control system. L. H. Schindel, USA

2832. Broadbent, D., The stability of time-error governed turbines in power systems, *Austral. J. appl. Sci.* **6,** 3, 281–287, Sept. 1955.

Paper is third of a series by author. First paper ["Integral governing of turbo alternators," *Elect. Engr. and Merchandiser* **29,** 11, 354–355, Feb. 1953] defines time-error governor as a servomechanism possessing one integration in the forward loop and whose input is standard time (output is electric time). The actuating error is thus the difference between standard time and electric time. Such a system will not only keep the electric frequency equal to the standard frequency but will also keep the total number of cycles at the output (electric time) equal to the total number of cycles at the input (standard time), after an initial correction for servo-tracking error. This is contrasted with speed governing in which only the frequencies are maintained equal and a time error (error in total number of cycles) may accumulate. In the latter system, time error is removed by periodic correction (either automatic or manual). This system is almost universally used because of its substantially lower cost and because periodic time correction is adequate.

In the second paper ["Stability of integral governing," ibid, **32,** 2, 40–42, May 1955], author develops simple equations for time-error governing and speed governing.

In this third paper, which constitutes an appendix to the two previous ones, author describes analog designed to represent equation for time-error-governed system and provides several solutions for cases of isolated and parallel machines.

S. Z. Dushkes, USA

2833. Davidson, G. M., and Nashman, L., How stabilization improves closed-loop operation, *Control Engng.* **2,** 12, 67–74, Dec. 1955.

2834. Meiller, F., and Reynaud, F., Transmission fluids in aeronautics (in French), *ONERA NT* no. 28. 42 pp., 1955.

Vibrations, Balancing

(See also Revs. 2814, 2815, 2816, 2828, 2878, 2946, 3097, 3113, 3120)

2835. Berry, J. G., and Naghdi, P. M., On the vibration of elastic bodies having time-dependent boundary conditions, *Quart. appl. Math.* **14,** 1, 43–50, Apr. 1956.

Title problem is recast into one with homogeneous boundary conditions and additional body forces. A formal solution of the equations of the linear theory of elasticity is given in terms of the characteristic functions (assumed known) of the homogeneous problem. This is a generalization, to three dimensions, of a classical method applied recently by Mindlin and Goodman to the Bernoulli-Euler beam equation [AMR **4,** Rev. 1926] and by Herrmann to longitudinal vibrations of rods [AMR **7,** Rev. 3817] and flexural vibrations of Timoshenko beams [AMR **8,** Rev. 2262]. A solution of the Timoshenko beam equation, obtained previously by Leonard and Budiansky by Laplace transform [AMR **6,** Rev. 2704], is given as an example. R. D. Mindlin, USA

2836. Van den Dungen, F. H., Rayleigh's principle in the case of damped oscillations (in French), *Acad. roy. Belgique, Bull. Cl. Sci.* (5) **40,** 11, 1038–1045, 1954.

Author deals with nonreal first eigenvalue r of fourth-order differential equation associated with the damped bending vibration of a nonuniform beam, its ends $x = 0$ and $x = 1$ being fixed and free from moments.

Improving of a guess r_0 is accomplished in usual way by constructing a solution with r_0 satisfying the two boundary conditions at $x = 0$ but only one at $x = 1$. Newton's regula falsi with respect to last boundary value yields a quadratic generalizing Rayleigh's equation for r_1.

Equivalence of this method with Rayleigh's in undamped case had been demonstrated previously [AMR **6,** Rev. 1811].

G. Plato, Germany

2837. Cheng, C.-M., Flexure vibration of uniform beams simply supported at equal intervals—normal modes and frequencies, *Cal. Inst. Technol. Guggenheim Aero. Lab.* OSR-TN-55-235, 10 pp. + 4 figs., July 1955.

Normal modes and frequencies are determined for a system with an arbitrary number of spans. A transcendental frequency equation is derived, and the frequency spectrum is examined for the general case. Mode shapes are expressed in closed form by means of elementary functions. A. I. Bellin, USA

2838. Grigolyuk, E. I., Nonlinear vibration and stability of slanting rods and shells (in Russian), *Izv. Akad. Nauk SSSR Otd. tekh. Nauk* no. 3, 33–68, Mar. 1955.

Free and forced nonlinear vibration with finite amplitudes and stability of a slender beam with initial curvature loaded with constant axial force are considered. The longitudinal displacements are assumed to be so small as to manifest themselves in the potential energy of the system only. This assumption is correct when the axial force is sufficiently smaller than the critical Euler's load and when the initial curvature is not too large.

The problem is solved under various end conditions corresponding to simply supported or clamped ends. In the first case, the Fourier expansion of the deflection curve is applied; in the second case, the approximation contains two trigonometric members. As a special case, results for a straight beam with longitudinally fixed ends can be obtained. Paper contains, moreover, the usual analysis of singular points in the phase plane.

In the second part, free and forced nonlinear vibrations of conical and spherical shells under analogical assumptions are taken into consideration. From these results some formulas for the frequency of plates are derived. Since the deflection of the plate is considered to be extensible, the vibration in this case is nonlinear also. K. Julis, Czechoslovakia

2839. Kennard, E. H., Forced vibrations of beams and the effect of sprung masses, *Inter. Shipbldg. Progr.* **2,** 16, 567–572, 1955.

Paper is the same as *David W. Taylor Mod. Basin Rep.* 955, June 1955. Author applies impedance methods to analysis of title

problem. He includes viscous and structural damping. Method demonstrates effect of mass and frequency variation of sprung mass on response of beam. Graphs demonstrate separation of frequency spectrum shown analytically by Rayleigh. Author considers effects of internal damping on forced beam vibration and shows that large damping restricts motion to vicinity of force. Several minor misprints in both versions—v not v, and $b = vl$ not $b + vl$ just before Eq. 8. R. Plunkett, USA

2840. Adamson, B., A method for measuring damping and frequencies of high modes of vibration of beams, *Publ. int. Assn. Bridge struct. Engng.* 16 pp., 1955.

Two experimental methods for determining natural frequencies of high modes of vibration of beams are proposed. The use of both methods simultaneously permits determination of damping as well.

The first method makes use of a load distribution similar to the n-th mode of vibration. If this loading is applied simultaneously and in transient form, the beam vibrates according to the n-th mode. Detonating charges exploded about one centimeter from the beam are used to accomplish this. The second method requires that two series-coupled strain gages be cemented to the beam at places which result in the maximum output of certain modes. A chart depicting the appropriate locations for uneven modes 1–17 is presented.

The uneven modes from 1 to 17 are studied for the case of a simply supported beam. It is found that Mindlin's "constant Q theory" best expresses damping in these beams. It is also concluded that Timoshenko's theory involving rotational inertia and shearing forces comes closest to agreeing with the measured frequencies. J. P. Vidosic, USA

2841. Shteinvol'f, L. I., Vibration proofing of dynamically unbalanced machines mounted on elastic beams (in Russian), *Trudi Inst. Mashinoved.* **15,** 57, 38–46, 1955.

A case of an unbalanced machine (with vertical harmonic force) suspended on an intermediate frame which is suspended from a rigidly mounted ceiling beam by two springs is investigated as a system of two degrees of freedom with forced vibration. Damping is assumed to be zero.

The displacement formulas are developed for all vibrating parts and, with the help of the method of A. H. Krilov, for elastic curve of ceiling beam. A practical suspension case is calculated in three ways: (1) with developed formulas; (2) assuming that frame is suspended on an equivalent spring; (3) reducing two-mass system to equivalent one-mass system.

As is self-evident, author finds that third case gives too high beam deflection; in second case, the error is small and simplifying calculations do not permit the finding of location of the beam section with the highest deflection. A. L. Nasvytis, USA

2842. Bagchi, R. N., and Chinmayee, D. (Mrs.), The experimental study of bowed strings and verification of the formula for the bowing pressure, *Indian J. theor. Phys.* **3,** 1, 1–16, Mar. 1955.

The paper deals with a quantitative verification of Kar's formula of a bowed string, *viz.*

$$P_{min} = \alpha\mu_D[l/d\,(V - V_0) + V_0] + P_0$$

where P_{min} is minimum bowing pressure to excite a steady vibration of the string, μ_D is the dynamical coefficient of friction during backward motion of the string, α a constant of proportion, l the length of the vibrating string, d the bowing distance, V the bowing velocity, $V - V_0$ the velocity of the string at the midpoint of the bowed region, and P_0 the additional pressure term due to resonance, statical displacement, and other factors. To determine V_0 of the formula, the variation of exact position of the zero-point with different distances and velocities is thoroughly studied by photographic method. From a set of values of P_{min} at different bowing distances for a given frequency and bowing velocity, $\alpha\mu_D$ and P_0 are calculated. These calculations have been repeated for two different velocities and frequencies.

From authors' summary

2843. Stephenson, C. V., Radial vibrations in short, hollow cylinders of barium titanate, *J. acoust. Soc. Amer.* **28,** 1, 51–56, Jan. 1956.

Paper deals with radial vibrations in hollow cylinders of electrostrictive ceramics of length small compared to its outer diameter. An expression for the radial coupling coefficient has been derived. Experiments have been performed with barium titanate elements. Experimentally obtained coupling coefficients are in close agreement with calculated values. It will be interesting, however, to verify the theoretical results in the case of other electrostrictive ceramics of various shapes. G. S. Verma, India

2844. Yamada, K., Vibration of turbine disc in its plane, *Proc. 2nd Japan nat. Congr. appl. Mech.*, 1952; Nat. Committee for Theor. appl. Mech., May 1953, 343–347.

Wave Motion in Solids, Impact

(See also Revs. 2814, 3020)

2845. O'Keefe, J., The initial value problem for the wave equation in the distributions of Schwartz, *Quart. J. Mech. appl. Math.* **8,** 4, 422–434, Dec. 1955.

Author constructs general solution to the wave equation using the theory of distributions. "Elementary solutions" to the wave equation are developed for n dimensions using Fourier transforms. These are generalized using the convolution theorem [L. Schwartz, "Theorie des Distributions"] to give the general solution to the initial-value problem. Solution to the nonhomogeneous wave equation is also given.

A review of classical solutions to the wave equation is included. R. E. Beckett, USA

2846. Greenspon, J. E., Stresses and deflections in flat rectangular plates under dynamic lateral loads based on linear theory, *David W. Taylor Mod. Basin Rep.* 774, 23 pp., Apr. 1955.

Paper treats response to uniformly distributed impulsive loads based on the mode-superposition method. The actual plate vibrational modes are approximated by products of beam normal functions. For simplicity, all vibratory modes are assumed to be in phase.

Approximate design formulas with tabulated values of parameters are given for simply supported and for clamped rectangular plates, using four symmetric modes. Load factors for several different pulse shapes are included. Data based on first mode only are given for plates having combinations of supported and clamped edges. D. Young, USA

2847. Huth, J. H., and Cole, J. D., Elastic-stress waves produced by pressure loads on a spherical shell, *J. appl. Mech.* **22,** 4, 473–478, Dec. 1955.

The blast wave is idealized as a plane wave carrying a pressure jump which moves across the sphere at a constant velocity. The effect of the sphere on this wave is neglected. The membrane shell theory is used to develop traveling-wave and mode solutions, and numerous graphs presenting the time dependence of stresses are given. Some of the conclusions reached on the basis of this analysis state that, as the velocity of the pressure wave increases without limit, the stresses approach those that would result from suddenly applying the pressure uniformly over the entire shell. For this limiting case the peak stress will be twice the static stress in compression and no tensile stresses arise. At

lower velocities, larger stresses of both signs result and may become four times the static stresses.

G. Herrmann, USA

2848. Naghdi, P. M., and Cooper, R. M., Propagation of elastic waves in cylindrical shells, including the effects of transverse shear and rotatory inertia, *J. acoust. Soc. Amer.* **28,** 1, 56–63, Jan. 1956.

Authors discuss the axisymmetric propagation of waves in their elastic cylindrical shells, using a simplified set of equations of motion but retaining the effects of both transverse shear and rotatory inertia. The simplified equations are such that, in the absence of shear and rotatory inertia, they reduce to those given by Donnell rather than to the more elaborate set known as Love's first approximation. It is shown that the loss of accuracy consequent on the use of the simplified version is negligibly small.

D. Williams, England

Elasticity Theory

(See also Revs. 2835, 2847, 2865, 2866, 2871, 2872, 2878, 2881, 2886, 2894, 2895, 2926, 2934, 3064)

Book—2849. Sokolnikoff, I. S., Mathematical theory of elasticity, 2nd ed., New York, McGraw-Hill Book Co., Inc., 1956, xi + 476 pp. $9.50.

This is the second edition of a distinguished text, first printed in 1946, now practically written anew. Headed by a historical sketch, chapter titles read: 1. Analysis of strain; 2. Analysis of stress; 3. Equations of elasticity; 4. Extension, torsion, and flexure of beams; 5. Two-dimensional elasto-static problems; 6. Three-dimensional problems; 7. Variational methods. Of these, the historical introduction and chaps. 5 and 6 are entirely new and, moreover, important additions, as compared with first edition, have been made to chaps. 4 and 7 so as to include a fairly complete exposition of modern approximate methods. Thus total number of pages has been increased from 373 to 476.

Characteristic of book is mathematically rigorous presentation in modern tensorial form, whenever practical, without losing contact with engineering outlook. Reader presumably being supposed to have but a fair knowledge of ordinary second-year calculus, intermissions of highly mathematical character have been necessary, including, e. g., complex-variable theory and conformal mapping.

No book of this size can pretend to be exhaustive on its subject. Author has chosen to entirely exclude plate and shell theory from this volume. Instead, there is an abundance of problems and ample references for collateral reading. The ambitious program leads right up to modern development, with references particularly to contemporary Russian works.

To reviewer, this excellent book seems to incorporate the best qualities of such outstanding texts as those of A. E. H. Love and S. Timoshenko.

F. K. G. Odqvist, Sweden

Book—2850. Lur'e, A. I., Three-dimensional problems in the theory of elasticity [Prostranstvenniye zadachi teorii uprugosti], Moscow, Gos. Izdat. Tekh.-Teor. Lit., 1955, 491 pp., 17 r/ 60 k.

This is a brilliant book on solving many famous classical problems of the mathematical theory of elasticity. It presupposes perfect familiarity with ordinary works (Timoshenko, Leibenson, etc.) and a reasonable working knowledge in advanced subjects of mathematics (e.g., vector and tensor analysis, partial differential equations, spherical and Bessel functions, Fourier integrals, etc.). A serious study of the volume shows a fine piece of research work.

Separate chapters of the book are written as complete units and may be studied (with the exception of some few topics) independently of each other. Each section concludes with a fine outline concerning literary data and historical development of pertinent questions.

Content is divided into 8 chapters with 69 subtopics. The first 60 pages are devoted to basic equations of elasticity. Nevertheless, one finds here not at all the usual comparatively simple deductions as known from the current textbooks, but an excellent mathematical foundation established in a very elegant way by means of advanced methods of modern vector and tensor analysis. Reader's attention is expressly called to the paragraph on solving the Pankovich-Neuber form of the equilibrium equations. The attractive second chapter deals in its 10 subsections with important classical problems concerning the elastic infinite and semi-infinite solid.

Of considerable importance is the following section about equilibrium of an infinite elastic plate (i.e., the plane elastic layer). A brilliant symbolic method of solving problems and masterly applications of complicated Fourier integrals brings enjoyment to everyone who likes excellent mathematical achievements. Fourth main part is directly connected with the preceding one and has for its subject the elastic equilibrium of thick plates. Fifth chapter is devoted to various questions concerning the mutual contact of elastic solids. Treated first are the famous Boussinesq problems for bodies of various shapes and also, more generally, the classical Hertz problem which forms the starting point in the theory of contact in elasticity.

Sixth main part treats the deformation of an elastic sphere under symmetrical load, whereas the following chapter deals in its 9 subsections with the analogous questions for the case of a circular cylinder. Author illustrates the excellent contribution of the Russian savants to solving extremely difficult questions of this kind. The last section treats the general equilibrium problem of elastic spheres.

Volume concludes with a nominal and a subject index. There are many numerical results and tables throughout the text. The presentation is illustrated by 35 figures; paper is of first-class quality and print is excellent. The study of this brilliant book is hard work, but it will bring a rich reward to every mathematician, engineer, and physicist.

V. Vodicka, Czechoslovakia

2851. Hashin, Z., The moduli of an elastic solid reinforced by rigid particles, *Bull. Res. Counc. Israel* **5C,** 1, 46–59, Dec. 1955.

Author considers the effect on the moduli of an elastic solid of adding particles of infinite rigidity. He assumes that the solid is an ideal elastic, isotropic, and homogeneous material; that the particles are spherical, perfectly rigid, and dispersed at random; that there is perfect adhesion between particles and solid; and that the volume concentration of the particles is small, i.e., less than 3%.

An extensive mathematical analysis leads to the following results

$$\kappa^*/\kappa = 1 + 3c_v\,(1-\nu)/(1+\nu) \qquad [1]$$

$$G^*/G = 1 + 15c_v\,(1-\nu)/2(4-5\nu) \qquad [2]$$

where

κ = bulk modulus of original solid
G = shear " " " "
ν = Poisson's ratio of " "
c_v = volume concentration of the particles

and the quantities having asterisks refer to the combined system of original solid plus particles.

An interesting discussion is included in which the analogy between an incompressible elastic medium and a viscous fluid in steady slow motion is pointed out. Thus, substitution of $\nu = \frac{1}{2}$ in [2] above leads to

$$G^*/G = 1 + 2.5\,c_v \qquad [3]$$

an equation originally derived by A. Einstein [*Ann. Phys.* **34,** 591, 1911] for the relationship between the viscosity of a solution

G^* and the viscosity of a medium (G), having a volume concentration c_v of a solute. Discussion concludes with reference to the need for further theoretical work to cover a larger range of c_v and to allow for finite rigidity of the particles.

Paper should be of some interest to metallurgists as well as rheologists. J. M. Alexander, England

2852. Truesdell, C., The simplest rate theory of pure elasticity, *Comm. pure appl. Math.* **8,** 1, 123–132, Feb. 1955.

Author formulates "simplest" theory of type: *rate of stress* = *f* (*rate of deformation*), obtains exact solutions, taking into account inertial forces, for certain homogeneous deformations, and establishes an existence and uniqueness theorem for pure homogeneous deformation. He concludes from comparing exact solutions with corresponding solutions obtained neglecting inertia that the neglect can lead to serious errors in situations generally regarded as quasi-static. In the formulation, he notes reasons why it is improper to take "rate of stress" to be material derivative of stress and introduces a more suitable definition.

J. L. Ericksen, USA

2853. Truesdell, C., Hypo-elasticity, *J. rational Mech. Analysis* **4,** 1, 83–133, 1955.

Author formulates a general theory of the type *stress rate* = *f* (*stress, rate of deformation*) for isotropic media. If no modulus dimensionally independent of stress occurs, the material is called hypoelastic. For these, *f* is linear and homogeneous in the rate of deformation. Many plasticity equations are of this type, but involve improperly defined stress rate, whereas the author's equations do not. Several exact solutions, including hydrostatic stress, simple shear, simple extension, and torsion are derived and discussed in detail, inertial effects being taken into account. Here most attention is given to cases where *f* is independent of, or linear in, the stress. Even for these cases, it is virtually impossible to briefly summarize the variety of possible types of behavior. An existence and uniqueness theorem for homogeneous stress is established.

The general flavor is that of a somewhat tentative, path-breaking memoir. Without implying a paucity of concrete results, reviewer feels that this work is of most value as a stimulus to further fundamental research in this area. Unanswered and partially answered questions abound. In particular, doubt which author casts on customary practice of neglecting inertia demands consideration. J. L. Ericksen, USA

2854. Il'yushin, A. A., Relationships between stresses and small deformations in the mechanics of solid media (in Russian), *Prikl. Mat. Mekh.* **18,** 6, 641–666, 1954.

By utilizing certain tensor invariants constructed from the stress and strain tensors for a general elastic-plastic media, author derives basic stress-strain relationships for small deformations. Then, by introducing a suitable metric, various geometrical relations are derived and employed in the above. The results are simplified in the isotropic case, as is to be expected, and author considers the problem of a system under plane stress. The work is an extension of author's earlier work in this field and in the present paper he exploits the methods found in E. Cartan's theory of SPINORS. Paper is considered fundamental in approach but not foundational. J. J. Brandstatter, USA

2855. Nowinski, J., Note on the definition of homogeneity and heterogeneity of elastic curvilinearly anisotropic bodies, *Bull. Acad. Polonaise Sci. (IV)* **3,** 3, 135–138, 1955.

2856. Kaestner, P. T., Compensating for thermal expansion, *Prod. Engng.* **27,** 3, 162–165, Mar. 1956.

How to calculate thermal effects on size and alignment of gears, bearings, and optical systems; methods of compensating for these effects. From author's summary

2857. Jasper, N. H., Temperature-induced stresses in beams and ships, *David W. Taylor Mod. Basin Rep.* 937, 25 pp., June 1955.

One (axial)-dimensional thermal stresses are computed for non-homogeneous hollow beams of uniform cross section and temperature distribution along the length. Usual method of superimposing stress to prevent thermal expansion with stresses to restore zero net axial force and transverse moments over the section is used. Simultaneous measurement of stress and temperature variations on full-scale ships is recommended. M. L. Williams, USA

2858. Tremmel, E., Contribution to the problem of thermal stresses in disks (in German), *Ing.-Arch.* **23,** 3, 159–171, 1955.

General theory of thermal stress distribution in two-dimensional fields is given [compare author's contribution reviewed in AMR **9,** Rev. 400]. Using bipolar coordinates, author applies theory to the problem of a circular tube with eccentric circular hole. Temperature distribution is independent of time; temperature is constant along inner and along outer boundary. Final stress formulas are used to study thermal stresses in a half-plane with a circular hole, a problem which can be important in foundation theory. A numerical example shows that in case of a concrete foundation the influence of thermal stresses cannot be disregarded.

W. L. Esmeijer, Holland

2859. Abbassi, M. M., Simple solutions of Saint-Venant torsion problem by using Tchebycheff polynomials, *Quart. appl. Math.* **14,** 1, 75–81, Apr. 1956.

Using the well-known stress function form for the solution of the torsion problem in polar coordinates, author obtains the general stress function in terms of Tchebycheff polynomials. A portion of this general solution, containing undetermined constants, which corresponds to the equation of the boundary of the rod, is then matched to various simple boundaries to give solutions to these problems. The cross sections solved are circular cross section, elliptic cross section, equilateral triangular cross section, and a specially notched circular cross section.

S. F. Borg, USA

2860. Ramakanth, J., Finite torsion of aeolotropic and composite cylinders 1, *ZAMM* **35,** 12, 453–459, Dec. 1955.

Paper deals with the problem of finite torsion of a hollow right circular cylinder. The material has hexagonal type of aeolotropy. Substituting the components of finite displacements into the strain components transformed in cylindrical coordinate system and using linear stress-strain relations, stresses are obtained in relation to β, where β is a function of the radius r. β is determined by the equilibrium conditions. The established differential equation is solved by series. The constants of the series are determined by the boundary conditions. Values of the constants are computed for Beryl, which is a hexagonal crystal. As a special case, the solution for an isotropic material is treated.

F. Schultz-Grunow, Germany

2861. Sherman, D. I., Torsion of a circular cylinder reinforced by an elliptical rod (in Russian), *Inzhener. Sbornik, Akad. Nauk SSSR* **21,** 79–96, 1955.

Author analyzes the stress distribution over a cross section of twisted circular cylinder reinforced by a symmetrically situated elliptical rod, the material of which possesses other elastic qualities. At the same time, the resultant torsional rigidity of such a rod is estimated. In general, the applied method is based on Muskhelishvili's famous work on plane elasticity [AMR **4,** Rev. 577]. Nevertheless, it contains new valuable ideas, explained in author's previous papers. It must be mentioned that without referring to these papers, the theoretical analysis cannot be completely followed.

By means of presented method the problem is reduced to an

infinite system of regular algebraic equations, having some special qualities.

It follows from the numerical example that, owing to the arrangement described, maximum stresses can be somewhat reduced.

K. Julis, Czechoslovakia

2862. Amen-zade, Yu. A., Bending of a circular prismatic beam with an elliptical cavity (in Russian), *Inzhener. Sbornik, Akad. Nauk SSSR* **21**, 97–112, 1955.

The weakened cross section has two axes of symmetry. End conditions of the beam are assumed as follows: one edge of the beam clamped, the second one free and centrally loaded by a lateral force in the direction parallel to the little axis of the ellipse.

Use of the method developed by Sherman [see preceding review] allows the determination of stresses in each point of the cross section. Paper contains one numerical example.

K. Julis, Czechoslovakia

2863. Savel'ev, L. I., Characteristics of limiting cycles in true stresses (in Russian), *Vestnik Mash.* **35**, 2, 14–17, Feb. 1955.

Experimental Stress Analysis

(See also Revs. 2829, 2895, 3044)

2864. Leven, M. M., Quantitative three-dimensional photoelasticity, *Proc. Soc. exp. Stress Anal.* **12**, 2, 157–172, 1955.

A good study on the methods of three-dimensional photoelasticity, comparing the merits of normal slices and oblique incidence with tangential or "skin" slices and normal incidence. Author discusses the relative value of Tardy and Babinet compensators with or without photoelectric null-indicator. A shaft with circumferential groove subject to torsion and a shaft with transverse hole in tension serve to illustrate the methods. There is a remarkable evaluation of the accuracy of the procedures. Author does not think that better accuracy than ±5% is possible in the best available techniques. D. Vasarhelyi, USA

2865. Schumann, W., Theoretical and experimental investigations of the Saint Venant principle with special regard to plate theory (in German), *ETH* Prom. Nr. 2420, 84 pp., 1955.

Dissertation contains an extensive review of the Saint-Venant principle for a variety of two- and three-dimensional problems from both an applied and a fundamental point of view. Proofs, statements, and restatements from Saint Venant, Southwell, Goodier, Erim, Mises, Wick, and Sternberg are discussed, and the progress which has been made is described. Major original contribution is the experimental determination of Green's function, or rather its derivatives for plates in bending under transverse load. Photoelastic technique of Favre and Gilg is employed and the careful attention to elimination of error and precision of measurement associated with Favre and his students is clearly evident. A composite plate of CR 39 and glass stuck together with Marco 69c enables use of a transmission polariscope. A doublet, triplet, and double doublet transverse loading are applied to find the first and second derivatives of the Green's function.

D. C. Drucker, USA

2866. Leeman, E. R., Stresses in a circular ring. Analytical solutions compared with photoelestic evidence, *Engineering, Lond.* **181**, 4701, 201–203, Apr. 1956.

Paper describes a photoelastic investigation of stresses in flat circular rings subjected to diametral tensile loads applied to the internal boundary. Comparisons are drawn between known exact and approximate elastic analyses, and experimental results are shown to agree with Filon's exact solution. Reviewer comments that accuracy of the approximate solution for a complete ring obtained by joining half-rings depends on the ring's proportions and is less general than paper implies. Misprint occurs in Eq. [1], which should read $M_0 = (Wb/\pi)\{(\frac{1}{2}) - (\log_e 1/k)/(1 - 1/k^2)\}$.

L. Maunder, USA

2867. Racké, H. H., A simple photometric thickness-measurement method for the full evaluation of two-dimensional stresses on photoelastic models (in German), *ZVDI* **98**, 5, 165–170, Feb. 1956.

With normal photoelastic methods, the directions and the difference of the principal stresses can be determined in each point of a two-dimensional model. The change in thickness of the model due to the loading is a measure for the sum of the principal stresses, so knowledge of its value enables determination of the magnitude of both principal stresses. Author demonstrates the use of two parallel plates of glass, between which the model is placed, while the space between the model and plates is filled with a light-absorbing liquid (aqueous solution of potassium permanganate). The intensity of the passing light is a measure for the model thickness and can be determined by normal photometric methods, either directly with a photocell or by photographing the picture. To obtain sufficient accuracy, the thickness changes should be in the order of a few per cent; this is the case with frozen-stress models. Models loaded at room-temperature should be of a material with a low modulus of elasticity (50 kg/cm²) to obtain the required thickness changes. The material Vulkollan is recommended. The method is simple and inexpensive and seems to have advantages over the older methods of measuring the thickness change by means of mechanical devices. R. G. Boiten, Holland

2868. Cadambe, V., and Kaul, R. K., Some optical considerations in the design of a polariscope and in photoelastic stress analysis, *J. sci. indust. Res., India* **14B**, 2, 41–50, 1955.

The design of a polariscope incorporating features essential for the reproduction of fine fringe patterns in homogeneous stress fields, and also in nonhomogeneous fields including regions of high stress gradients, is described. The method of counting fringe orders accurately and the design of a photometer and its use are described. Correction factors necessary for taking into account various errors due to small obliquity of incident rays and optical unbalance are indicated. The details of design and construction of the polariscope built in the laboratory are also described.

From authors' summary

2869. Szecsi, A., Computation of beam structures. Parts I, II (in French), *Bull. tech. Suisse Rom.* **82**, 3, 29–35, Feb. 1956; 4, 45–52, Feb. 1956.

Author bases his method of beam-dimensioning on determination of effective elastic and plastic deformation of structures. Considering especially the actuation of bending moments and supposing the usual simplified strain-stress diagram, he derives for arbitrary shaped beams a formula for relative rotation of two sections, if stresses exceed elasticity limit of material. Obtained rotation enables the computation of statically indeterminate structures by successive approximation process. Author considers also the influence of unloading and analyzes remaining stresses in the structure caused by self-modification of Poisson's ratio.

He gives a detailed description of tests using thermal-treated steel bars, which confirm results of presented theory. Further tests with alternate loading of bars, proves elevation of elasticity limit if stresses surpass slightly the limit of proportionality.

Reviewer points out that developed theory is very simple and useful, recalling importance of consideration of longitudinal ex-

tension of plastic zone in structures. Computation assuming flow articulations ("Rotule" plastique) does not correspond with real behavior of structure when stresses exceed limit of elasticity.

H. Beer, Austria

2870. Anonymous, Barium titanate accelerometers and strain gauges, *Engineer, Lond.* **201,** 5230, 382–383, Apr. 1956.

Rods, Beams, Cables, Machine Elements

(See also Revs. 2813, 2822, 2825, 2836, 2837, 2838, 2839, 2840, 2850, 2857, 2861, 2866, 2869, 2884, 2885, 2886, 2888, 2897, 2901, 2903, 2904, 2932, 3143)

2871. Suzuki, S., On the decrease of flexural rigidity of beam due to elastic clamping, *J. mech. Lab., Tokyo* **1,** 2, 61–62, 1955.

2872. Godfrey, D. E. R., Tapered beams under distributed flank loads, *Aircr. Engng.* **26,** 306, 240–243, Aug. 1954.

Stevenson's complex potential method is applied to several problems of a wedge in a state of generalized plane stress. Loadings considered include concentrated force and couple at tip, shear and normal stress along one edge varying as an arbitrary power of distance from tip, and uniform body force. Corresponding solutions derived from Airy's stress function are, of course, well known. Author presents numerical results for two cases.

R. E. Newton, USA

2873. Ilyushin, A. A., Normal and tangential stress in the pure bending of beams beyond the limit of resiliency, and the analogy to the bending plates (in Russian), *Inzhener. Sbornik, Akad. Nauk SSSR* **19,** 3–12, 1954.

Author considers the pure bending problem beyond the limit of resiliency, because the Poisson's ratio increases very rapidly beyond this limit.

The basic equations are investigated for a beam whose cross section has two axes of symmetry, under assumption that a stress-strain curve is approximated with a broken straight line. The problem then reduces to the integration of the differential equation

$$(\partial/\partial x^2)[(e_i/\sigma_i)\phi_{xx} - Z_z] + (\partial^2/\partial y^2)[(e_i/\sigma_i)(\phi_{yy} - Z_1) + 2(\partial^2/\partial x \partial y)(e_i/\sigma_i)\phi_{xy}] = 0$$

where ϕ is the stress function which satisfies the boundary conditions and e_i/σ_i, Z_z are functions of ϕ.

An approximative solution is obtained by means of a small parameter (the coefficient of the volume compression) taking into consideration the second approximation. The supplementary stresses, caused by the material's compression, are determined also.

It is shown that this problem is analogous to the plate-bending problem.

The rate of the approximation's accuracy is indicated. The first approximation gives the error to 10% in the rigidity's increase direction, but the second only to 1% in opposite direction.

One example is described and the results are discussed.

D. Raškovič, Yugoslavia

2874. Chronowicz, A., The design of non-prismatic beams, *Concr. constr. Engng.* **51,** 5, 359–364, May 1956.

2875. Parkes, E. W., Stresses in boiler tubes, routine procedure suitable for desk calculating machine, *Engineering* **181,** 4695, 84–87. Jan. 1956.

Analysis and suggested procedure are similar to those conventionally employed in piping flexibility analysis, using the so-called "square-corner" approximation, in which a circular section of pipe is replaced by two or more straight sections for purposes of simplifying the calculations. Author shows in appendix that little error is likely to be involved in this replacement technique. Reviewer agrees with argument when tubes are not thin-walled. However, analysis does not take into account phenomenon of flattening of the tubular cross section, in the bending of circular arcs of pipe [AMR **9,** Rev. 1765, (Chap. 3)]. This gives rise to an accentuated flexibility of curved pipe sections and a corresponding stress intensification, both of which must be accounted for under various piping codes (see, e.g., ASA Code for Pressure Piping.) For thin-walled tubing it is known that the square-corner approximation can be unreliable [AMR **9,** Rev. 1765(pp. 107–114)]. Reviewer feels that author should reinforce the arguments of his appendix by noting that extra care may be required in studying thin-walled boiler tubing.

J. L. Lubkin, USA

Plates, Disks, Shells, Membranes

(See also Revs. 2838, 2846, 2848, 2850, 2865, 2866, 2873, 2890, 2891, 2892, 2893, 2903, 2923)

Book—2876. Gol'denveiser, A. L., Theory of elastic thin shells [Teoria uprugikh tonkikh obolochek], Moscow, Gosud. Izdat. Tekh.-Teor. Lit., 1953, 544 pp. $5.88.

Author presents the most comprehensive treatment of the theory of thin shells published to date, with the main stress on the more mathematical aspects of the subject at the expense of those of applied nature which have received more attention in the past in the books by V. Z. Vlasov, V. V. Novoshilov, and A. I. Lur'e.

Part I (93 pages) gives the foundations of the theory and includes a chapter on the theory of surfaces. The development of the basic equations makes wide use of vector analysis and the notation due to A. E. Love. Part II (101 pages) presents the membrane theory as a special case of the more general theory. It is developed in great detail, having in mind that in the later sections approximate methods of solution are to use the membrane solutions as starting points. Part III (75 pages) deals with problems of circular cylindrical shells and their solution by Fourier series, with particular attention to edge effects. Part IV (120 pages) is devoted to shells of arbitrary shape. A large section of this part deals with the asymptotic solutions of partial differential equations with small parameters and forms the basis for the unified theory of approximate methods of solution given in Part V (138 pages), which contains illustrative examples.

References are almost exclusively to Russian work in the field and this fact may be considered a disadvantage in a book of this nature which, in all other respects, should be of considerable interest to the theoretician.

J. R. M. Radok, USA

2877. Dehousse, N. M., Circular cylindrical shells (in French), *Bull. Centre Étud. Constr. génie civ. Hyd. Fluviale* **7,** 3–118, 1955.

Composition of practical relations, tables, and curves for circular cylindrical shells, especially with regard to roof constructions. The theory is based on the work of Dishinger and Flügge and the further literature up to 1953.

H. Neuber, Germany

2878. Volterra, E. G., The equations of motion for elastic plates and cylindrical shells deduced by the use of the "Method of internal constraints," *Rensselaer Polytech. Inst. technol. Rep.* 3, 36 pp., Feb. 1956.

Taking x, y, z coordinates referred to three curvilinear orthogonal axes such that the x-y plane coincides with the middle surface of a cylindrical shell, x axis coincides with the line of principal curvature, and the z axis with the normal, author assumes that the

displacement vector $\bar{V}(x,y,z,t)$ is connected with two other vectors $\bar{\mu}(x,y,t)$ and $\bar{v}(x,y,t)$ by the relation

$$\bar{V}(x,y,z,t) = Z\bar{\mu}(x,y,t) + \bar{v}(x,y,t)$$

Expressions for kinetic and potential energies are then found on this assumption, and the equations of motion for cylindrical shells (taking into account the effects of shear and rotatory inertia) are derived by using Hamilton's principle.

The equations of motion for rectangular plates are deduced from those for the shell by making curvature tend to zero. Finally, the corresponding equations for circular plates are expressed in cylindrical coordinates. A. M. SenGupta, India

2879. Mansfield, E. H., and Fine, M., The effect of uniformly spaced flexible ribs on the stresses due to self-equilibrating systems applied to long thin-walled cylinders, *Aero. Res. Counc. Lond. Rep. Mem.* no. 2832, 43 pp., Aug. 1947, published 1954.

In many problems relating to the stressing of thin-walled cylinders, and in particular those concerned with the stresses set up in a cylinder under torsion when one section is restrained against warping, it has been commonly assumed that sections have their shape retained by closely spaced stiff ribs. Justification for this assumption is that, for certain types of loading, the ribs of most practical structures do little work in maintaining the section shape (and the analysis is considerably simplified).

In this report the effect of discrete, flexible ribs has been investigated and the results have been incorporated in a number of graphs which show the effect of rib-flexibility in a long thin-walled cylinder of arbitrary shape under end constraint. Some of the results of these investigations are, as would be expected, of a negative character, in that they show that for certain types of end conditions (roughly, those in which the predominating self-equilibrating loads act parallel to the cylinder axis) the effect of rib-flexibility is negligible. But rib-flexibility is of paramount importance when self-equilibrating shear-distorting forces are applied to a cylinder—such as occur at a wing cut-out or near an overhanging engine—and this report makes the stress distribution in such a case readily determinable.

It is shown that the complete stress die-away pattern depends, apart from the boundary conditions, on two nondimensional parameters. These parameters are functions of the type of end constraint as well as of the structure dimensions and elastic constants. Expressions are given for determining these parameters when the cylinder shape and loading are arbitrary.

The simplified case of a four-boom cylinder of rectangular section under torque is treated separately in a second appendix.

The solution is strictly true for a four-boom cylinder or when the self-equilibrating end-load system is orthogonal (eigenload); but as minimum-energy methods are used in the analysis, the results are believed to be substantially correct for a smoothly varying end-load system applied to a cylinder of arbitrary shape.

From authors' summary

2880. Riparbelli, C., Oblique bending of a rectangular plate, *Cornell aero. Lab. Rep.* OSR-TN-55-147, 21 pp. + 11 figs., May 1955.

Author has investigated the deformation characteristics of a thin rectangular plate and a thin trapezoidal plate under the action of equal and opposite moments whose vector forms an angle β with the normal to the longitudinal axis of the plate such that when β equals 0°, pure bending is applied to the plate, and when β equals 90°, pure torque is applied when referenced to the longitudinal axis of the plate. These investigations are made to assist the author in developing a method of analysis for thin plate-like wings, which will be presented in future papers. He points out that experimental evidence indicates that when a rectangular plate with free ends is subjected to oblique bending, the plate bends about a family of lines of maximum curvature which are also oblique and form an angle α with the normal to the plate's longitudinal axis. The lines of maximum curvature are the lines which would form in stress-coat when the plate is subjected to oblique bending.

In the case of rectangular plates, author assumes that the lines of maximum curvature are straight. Through the use of energy equations he develops the relation between the angle of the lines of maximum curvature α and the angle of the moment β. He finds that $\alpha = \beta/2$. The formula for the maximum curvature C is also developed.

The second part of the paper removes the restriction that the lines of maximum curvature be straight. With the use of tests on stress-coated specimens and further analytical work, author refines the solutions obtained in part one. He finds that at the centerline of the plate the angle of the line of maximum curvature α is ½ that of the angle of the applied moment β. At the edges of the plate $\alpha = \beta/4$. K. Arnstein, USA

2881. Crawford, R. F., A theory for the elastic deflections of plates integrally stiffened on one side, *NACA TN* 3646, 21 pp., Apr. 1956.

An elastic-deflection theory is presented for anisotropic plates which exhibit coupling between bending and stretching. In particular, the theory provides a basis for analyzing plates integrally stiffened on one side. The effect of coupling is responsible for the presence of added terms in the potential energy expression, the small- and large-deflection equations of equilibrium, and the boundary conditions. Example calculations show that, in the case of a simply supported square plate loaded in compression and having equal flexural stiffness in the principal directions, coupling may significantly lower the in-plane compressive load at which deflections grow rapidly.

From author's summary by D. Kececioglu, USA

2882. Rozenberg, L. A., On the pressure of a rigid body on a plate (in Russian), *Inzbener. Sbornik, Akad. Nauk SSSR* **21,** 151-155, 1955.

Author develops formulas for the determination of the distribution of the pressure on a circular plate when acted upon by cylindrical rigid body. Two cases are analyzed: (1) A thin circular plate freely supported around its edge; (2) the same clamped around the edge. It is shown that theoretically the pressure is sharply increased toward the outer part of the area of contact. When the area of contact is small, pressure is nearly uniformly distributed. In reality, however, the increase of pressure near the outer contact will be somewhat smaller due to the local crushing effect. This note is an improvement on L. A. Gallin's method showing the outline of the pressure area only [AMR **1,** Rev. 1322]

M. Maletz, USA

2883. German, D. Ya., and Prokopov, V. K., Bending of a sectoral plate with a fastened rim due to a uniformly distributed load (in Russian), *Inzbener. Sbornik, Akad. Nauk SSSR* **21,** 120-127, 1955.

Several terms of J. H. Michell's well-known general expression for biharmonic function in polar coordinates are applied, although author attributes them to K. A. Kitover and G. A. Grinberg. Boundary conditions at the clamped circular boundary are exactly satisfied. Author's assertion that they can be also satisfied at the free radial boundary with any accuracy required is not proved, not even confirmed by the numerical example which he gives.

J. M. Klitchieff, Yugoslavia

Buckling Problems

(See also Rev. 2915)

2884. **Goldberg, J. E., Bogdanoff, J. L., and Lo, H., Inelastic buckling of non-uniform columns,** *Proc. Amer. Soc. civ. Engrs.* 82, EM 2 (*J. Engng. Mech. Div.*), Pap. 943, 11 pp., Apr. 1956.

A simple numerical procedure is presented for calculating the critical loads of nonuniform pin-ended columns. The differential column equation is transformed into an integral equation for curvatures, which is solved numerically for trial values of the critical load. Inelastic behavior is accounted for by introducing a variable tangential modulus. Numerical examples are worked out.

P. Cicala, Italy

2885. **Hu, L. W., and Triner, N. H., Bending creep and its application to beam-columns,** *J. appl. Mech.* **23,** 1, 35–42, Mar. 1956.

Paper reports on the results of an experimental investigation of the creep behavior of beam columns of magnesium alloy FSI-F at 260 F.

A theoretical analysis of the situation is also given. It is shown that the deflection of the free end of the column can be estimated by means of a semigraphical procedure. Agreement between theory and experiment is quite good for three of the four cases reported.

Y.-H. Pao, USA

2886. **Zickel, J., Pretwisted beams and columns,** *J. appl. Mech.* 23, 2, 165–175, June 1956.

By assuming that the internal stresses consist of normal stresses, Saint Venant, and secondary shear stresses, author shows that the integrals that determine the forces and moments can be integrated for the doubly symmetric cruciform, I-beam, and rectangular cross sections. The external forces and moments and the axial displacement can be eliminated from the equilibrium equations leaving, as variables, the twist and the lateral displacements.

Solution of the equilibrium equations shows that a column of cruciform section with identical principal moments of inertia is weakened by pretwist. Columns with equal but not identical principal moments of inertia cannot be strengthened, but if the principal moments of inertia are unequal, then the column can be strengthened by pretwist. Strenghtening by pretwist is effective up to pretwists of 2π, but decreases thereafter–even below the Euler buckling load.

A 14 × 12 WF–84-lb column (ratio of principal moments of inertia is 4.06) is strengthened by pretwist by a factor of 1.5 maximum when the slenderness ratio is 200, and by a factor of 1.3 when the slenderness ratio is 50. There is no strengthening when the ratio of pretwist to slenderness ratio is more than 0.2.

A thin strip whose principal moments of inertia have a 400 to 1 ratio is strengthened by pretwist by a factor of 1.9 when the slenderness ratio is 2000, by a factor of 1.1 when the slenderness ratio is 200, but is weakened by pretwist if the slenderness ratio is 100. No strengthening results if the ratio of pretwist-to-slenderness ratio exceeds 0.3 for thin strips whose slenderness ratio is between 200 and 2000.

N. C. Costakos, USA

2887. **Patel, S. A., Bloom, M., Erickson, B., Chwick, A., and Hoff, N. J., Development of equipment and of experimental techniques for column creep tests,** *NACA TN* 3493, 20 pp., Sept. 1955.

Equipment and procedures developed in order to test aluminum-alloy columns subjected to constant loads at elevated temperatures are described. The apparatus consists primarily of a lever type of loading mechanism and an electric oven. Particular emphasis was laid upon the determination of the influence of initial deviations from straightness upon the critical time of the column. Test results which were obtained with 15 2024-T4 aluminum-alloy columns having a slenderness ratio of 111 are presented. It was found that the critical time, that is, the time necessary for the column to buckle when subjected to a constant load, depends greatly upon the ratio of the applied load to the static critical load at the temperature of the test. The critical time is also a function of the initial deviation from straightness.

From authors' summary

2888. **Solvey, J., The influence of the torsion-bending constant on the lateral instability of beams in the elastic range,** *Aero. Res. Labs., Melbourne, Austral. Rep.* SM 231, 13 pp. + 1 table + 1 fig., Aug. 1955.

Approximate solution of problem is given using strain energy method. Ratios of critical loads to those for corresponding beams having negligible torsion-bending constant are given for various loadings and boundary conditions.

H. D. Conway, USA

2889. **Erickson, B., Nardo, S. V., and Hoff, N. J., Deviations from straightness of carefully machined and centered columns,** *Proc. Soc. exp. Stress. Anal.* **13,** 1, 67–76, 1955.

Paper describes a most careful technique devised for setting up simple column specimens for test, so as to render as small as possible any first harmonic in initial deflection which would arise from initial lack of straightness. In subsequent tests on six small columns, consistent results showed initial magnitude of first harmonic to be of order of one ten-thousandth of an inch.

Technique centers round use of electric resistance strain gages together with harmonic analysis of curvature distribution under load.

As mentioned, initial deviations from straightness of columns is an important factor in column characteristics as revealed by tests at high speed of loading or at elevated temperatures. Work on both these topics has been initiated by senior author, and present paper is relevant to that work.

F. S. Shaw, Australia

2890. **Mitchell, L. H., An experimental investigation of stress diffusion in non-buckling plates,** *Aero. Res. Counc. Lond. Rep. Mem.* 2878, 20 pp., 1955.

Purpose of this report is to provide experimental results for comparison with theoretical analyses of stress-diffusion problems. The structures considered consist of plane reinforced sheet which has been assumed not to buckle. Symmetrical loads are applied to the edge booms connected to the sheet by continuous no-slip joints. Attention is concentrated on the stress distribution near the ends of the parallel strips of plate.

An outline of the existing theoretical work which is applicable to this type of problem is given. The stringer-sheet theory, the only one capable of dealing adequately with unreinforced sheet, is compared with the photoelastic results.

It is shown that the stringer-sheet theory overestimates the peak shear stresses near the corners of the strips and consequently also the rate of diffusion of load from boom to sheet. It is also shown that the experimental shear stresses are in reasonable agreement with those predicted by a more exact plane-stress theory. This theory predicts that the peak shear stress in the plate is $2/\pi$ times the direct stress in the boom at the end of the panel. However, with the type of joint considered here, the maximum shear stress is likely to be much higher than the value given by this prediction.

Some attention is also given to transverse end stiffeners and it would seem that these normally have very little effect on the shear stresses.

The photoelastic models were made from the Allylstrene plastic called C.R.39. The no-slip joints were obtained by gluing the stiffeners to the plates.

From author's summary

Book—2891. Radok, J. R. M., The stability of stiffened plates and shells [Die Stabilität der versteiften Platten und Schalen], Groningen (Holland), P. Noordhoff N. V., 1955, 47 pp. $0.75 (paperbound).

Paper considers thin plates and shells in which the stiffeners are idealized as lines. Using infinite series, characteristic equations are obtained for the critical loads for flat plates and circular cylinders with stiffeners in either the longitudinal or lateral directions. Numerical results are presented for a rectangular plate with stiffeners perpendicular to the direction of load.

J. M. Gere, USA

2892. Schnell, W., Calculation of buckling loads of rectangular plates with longitudinal or transverse stiffeners under compression (in German), *ZAMM* **36,** 1/2, 36–51, Jan./Feb. 1956.

The plates considered are simply supported along two opposite sides and compressed by boundary loads along two opposite sides or all four sides. Their differential equation is transformed into a fourth-order ordinary differential equation by letting $u(x,y) = w(y) \sin n\pi x/a$. The general solution of this ordinary equation is written assuming as arbitrary constants the amplitudes of w, dw/dy, the bending moment, and the shear at the boundary $y = 0$. The general solution and its derivatives are then considered as matrix operators giving the amplitudes of these quantities at any other section y. The plate operators can be combined with the operators corresponding to transverse or longitudinal stiffeners to obtain the matrix operator corresponding to the stiffened plate.

The determinantal equation for the critical value of the loads expresses the conditions at the other edge $y = b$ and is of the second order. The method is extended to orthotropic plates and to the evaluation of the critical value of a compression varying from point to point of the boundary, by substitution of a step-wise constant compression.

The matrix-operator method allows the direct writing of the complicated transcendental buckling equations for an important but limited category of plates. It has been used in connection with other characteristic value problems by various authors.

M. G. Salvadori, USA

2893. Wittrick, W. H., Buckling of an infinite simply supported strip under combined longitudinal compression, transverse compression, bending and shear, *Aero. Res. Lab., Melbourne, Austral., Rep.* SM 234, 21 pp. + 21 figs., Sept. 1955.

Problem formulation by Galerkin's method leads to determinant of infinite order which is then approximated by a second- or third-order determinant. Solution yields exact results for two cases. Interaction curves are plotted for all combinations of the four basic stress systems with errors estimated at less than 4½%.

R. B. McCalley, Jr., US

2894. Korányi, I., Limit curves between the elastic and the plastic range of buckling of webs under banding and shear stress (in German), *Acta Techn. Hung. Budapest* **13,** 1/2, 61–68, 1956.

Joints and Joining Methods

(See also Revs. 2926, 2928)

2895. Masubuchi, K., Ogura, Y., Ishihara, Y., and Hoshino, J., Studies on the mechanism of the origin and the method of reducing the deformation of shell plating in welded ships, *Inter. Shipbldg. Progr.* **3,** 19, 123–133, Mar. 1956.

Paper reports on the deflections and stresses resulting from fillet welding of plates 12 mm thick perpendicular to a flat bottom plate 19 mm thick. Variables are span between stiffeners, size of fillet weld, penetration of weld, and freedom or constraint. Deformations of flat plate between stiffeners and stresses were measured. The work is being continued.

F. E. Reed, USA

2896. Rudy, J. F., McCauley, R. B., and Green, R. S., The behavior of spot welds under stress, *Welding J.* **35,** 2, 65-s—71-s, Feb. 1956.

Purpose of this paper is to give evidence of a relationship between four spot-welding parameters: (1) The physical strength of the spot weld in tension shear (coupon is in tension-weld in shear); (2) the mechanism of weld failure when pulled in tension shear; (3) the macro-geometry of the weld cross section; (4) "sectioned spot shear test" observations. These latter are observations of the exposed cross section of a spot weld as load is applied to the section in tension shear.

From authors' summary by H. O. Albano, Argentina

2897. Thamm, S., Calculation of shrink connections between cylindrical machine parts rotating at high speed (in German), *ZVDI* **98,** 11, 463–474, Apr. 1956

Solutions are first presented for the problem of a rotating assembly consisting of one parallel-sided disk shrunk onto another of the same axial length. Then consideration is given to a rotating assembly in which the outer member is a thin cylindrical shell of relatively great axial length, at least one end of which is shrunk onto a parallel-sided disk of very small axial length.

In the first problem, author uses the simple theory of parallel-sided disks as found in the usual textbooks on advanced strength of materials; the second problem requires, in addition, the familiar theory of the bending of cylindrical shells similar to the bending of beams on elastic foundations.

Solutions presented include combined effects of shrink-fit and centrifugal stresses. Results are involved but, reviewer believes, trivial and of limited application. Less simple geometries of broader application are readily solved by methods of superposition.

E. J. McBride, USA

2898. Doyle, L. E., Better, B. R., and Chao, B. T., A design for nonsticking plug and ring gages and locators, *Univ. Ill. Engng. Exp. Sta. Bull.* 433, 28 pp., Jan. 1956.

A close-fitting plug tends to stick when it enters a hole. The distance that the plug must enter before sticking ceases has been found to be the product of the diameter of the plug times the coefficient of friction. Sticking can be prevented by making the plug so short that diagonally opposite points on its diameter do not contact opposite sides of the hole, or by providing a means to lead the plug into the hole beyond the point where sticking ceases before full contact is made.

On the basis of the evaluation of the causes of sticking, formulas have been derived and are given for the proper design of common types of plugs and rings used for gages, fixtures, jigs, and other tools.

From authors' summary

2899. Winter, G., Tests on bolted connections in light gage steel, *Proc. Amer. Soc. civ. Engrs.* **82,** ST 2 (*J. Struct. Div.*), Pap. 920, 25 pp., Mar. 1956.

Results are summarized for 574 tests of bolted joints in light-gage steel, covering considerable ranges in bolt diameter, sheet thickness, mechanical properties of sheet and bolts, and edge distance. Joint failures fall into four classes: shear in sheet ahead of the bolt, bearing and "piling-up" the sheet ahead of the bolt, transverse tension-tearing of the sheet, and shear in the bolt. Formulas for predicting the various types of failure are developed which, when used with an adequate factor of safety, will limit the deformations under design loads to acceptable values.

M. Holt, USA

Structures

See also Revs. 2874, 2875, 2880, 2881, 2894, 2920, 2922, 2942, 2943, 2962, 3013, 3014, 3119, 3122, 3124, 3129, 3130, 3131)

Book—**2900. ACI-ASCE Comm. 323, Prestressed concrete (Bibliography no. 1), 2nd ed.,** Detroit, American Concrete Institute, 1955, 98 pp. + 2 patents. $2.

This bibliography was prepared by ACI-ASCE Committee 323 with members of international reputation, such as Abeles, Amirikian, Dobell, Freyssionet, Lin, Newmark, headed by Thor Germundsson, and is essential for any engineer interested in prestressed concrete. Second edition contains many additions, but still is not complete and some important papers, reports on improvements and patents, especially such of early dates, should be added in the next edition. For example, several articles by leading specialists, such as Magnel, Wettstein, and others, published in the monthly magazine *Hormigon elastico* devoted exclusively to prestressed concrete, are not mentioned. Karl Wettstein is one of the first inventors who was granted patents on prestressed concrete, 1924, and there is a great number of articles and research reports on his fundamental principles which are not included in the bibliography. In some of the articles published in the Journal of the ACI the writers of the pertinent discussions are not mentioned although their discussions are of importance. It is suggested by the writer to give the year of the granted patents.

The reviewer was the research engineer of Karl Wettstein's prestressed concrete structures since he started to build them in 1919, and published a number of his reports, as e.g. Technical Encyclopaedia, Vol. 15, pp. 335 and 336; *Journal of ACI*, Dec. 1950, pp. 724-1 to 4. J. J. Polivka, USA

2901. Britton, H. W., The flexural resistance of prestressed simply supported concrete beams within the elastic range, *J. Instn. Engrs. Austral.* **28,** 1-2, 39-46, Jan.-Feb. 1956.

Using the well-known approximate formulas of prestressed-concrete beam analysis, author develops a cut-and-try design procedure for certain assumed shapes.

The method, basically, satisfies four required stress conditions by setting up four simultaneous equations in terms of arbitrary "shape factors." The equations, when solved, lead to two separate cases, "short spans" and "long spans," depending upon steel eccentricity.

Design tables are included and two illustrative problems are solved. S. F. Borg, USA

2902. Cowan, H. J., The ultimate strength of prestressed concrete beams, *Struct. Engr.* **33,** 7, 197-212, July 1955.

Professor Cowan derives a set of ultimate strength formulas on the basis of assumed properties of concrete and steel, and with the aid of equilibrium and compatibility equations. For concrete, the classic elastoplastic diagram is employed; for steel, the stress-strain curve is assumed to consist of three straight lines, with a horizontal end portion. The following cases are analyzed: primary tension failure, primary compression failure of rectangular and flanged sections, influence of the prestressing steel near the compression face.

Professor Cowan has succeeded in finding a group of relatively simple expressions, containing the fundamental variables involved, and some empirical constants. It is of interest to remark that, for under-reinforced beams, the ultimate bending moment is expressed as a function of the steel area and the inner arm, and not as a function of the concrete stress block, as is usual in the current literature. This is theoretically more correct, because when a beam fails by steel rupture, the concrete does not reach its ultimate compressive strain, but, due to the low value of the neutral axis depth, the influence of the stress block form on the lever arm is small. This form of the ultimate bending formula has, from the practical point of view, the interesting feature of simplicity in use, as it is feasible to solve the design requirements with the least possible calculation.

Studying the graph representing the variation of the moment-arm coefficient with the steel ratio, and the ratio of ultimate steel strength to the concrete strength, it can be concluded that, in the range of the steel ratio from 0 to 0.5 and in the range of the ultimate strength ratio fron 20 to 40, it is possible to assume the inner arm as 9.95 of the effective depth, with an error not exceeding 6%, which is small compared with other uncertainties involved in the reinforced-concrete design. This simplified solution has been proposed by Y. Guyon, "Lé Béton Précontraint," Eyrolles, Paris 1951, and Magnel, "The ultimate strength of prestressed beams," *Concrete and Constructional Engineering*, Feb. 1953. The expression for the primary compression failure is more complicated, and, as the author indicates, it is not possible to reach the same accuracy as in the primary tension failure, due to the empirical constants involved and to the uncertainties with respect to the concrete strength.

C. A. Sciammarella, Argentina

2903. Link, H., Strength anaylsis of a steel-concrete compound shaft lining with single tubbing pile (in German), *Tech. Mitt. Krupp* **13,** 1, 10-20, Apr. 1955.

The lining of the shaft (374.5 m deep, inside diam 7.2 m) consists of single-walled rolled steel tubbings backed by concrete; steel and concrete are connected by steel web plates and ring strips so that a compound body is formed like reinforced concrete. The loads are water pressure and nonuniform rock pressure by which the ring of the shaft walling cross section is stressed by combined compression and bending.

The strength calculation includes in detail the stresses and buckling strength. Most interesting are the considerations about the naturally somewhat uncertain hypotheses of the nonuniformity of rock pressure. E. Seydel, Germany

2904. Viest, I. M., Investigation of stud shear connectors for composite concrete and steel T-beams, *J. Amer. Concr. Inst.* **27,** 8, 875-891, Apr. 1956.

Push-out tests of round steel studs were made to determine the behavior and load-carrying capacity of stud shear connectors. The tests have shown that a steel stud is suitable for use as a shear connector in composite concrete and steel construction. Empirical equations are presented for determining critical load.

From author's summary

2905. Steinman, D. B., Mackinac bridge - Designed for complete aerodynamic stability, *Civ. Engng., N.Y.* **26,** 5, 37-41, May 1956.

One of the longest steel bridges (26,195 ft) required the solution of many engineering problems, the most important being the research of aerodynamic stability (by Steinman), and the extremely deep foundations (by R. E. Davis and M. Polivka, professors of the University of California). After the collapse of the Tacoma Bridge, Dr. Steinman devoted 17 years of intensive investigation of the principles involved in aerodynamic stability, especially pertaining to vertical, torsional, and combined oscillation of bridges. These investigations were made for the Mackinac Bridge by simple model tests with the assistance of F. J. Maher of Virginia Polytechnic Institute. The tests proved that open grid over the full width of the roadway does not produce aerodynamic stability, and solid deck is needed for atmospheric damping, to act against oscillations by atmospheric resistance. The use of open floor-beams eliminates an important origin for the initiation of oscillations by gusts and quartering winds. Thorough aerodynamic investigation was conducted by F. B. Farquharson in his laboratory at the University of Washington, who predicted the failure of the Tacoma Bridge. He also tested the model with all roadway and sidewalk grids closed to represent the hypothetical condition of such openings closed by ice. It was concluded that the bridge

was absolutely safe, aerodynamically, even under the most adverse conditions. The bridge is being built for the Mackinac Bridge Authority. J. J. Polivka, USA

2906. Sattler, K., Considerations of the computation method of Guyon-Massonnet on free supporting beams and extension of the method to general systems (in German), *Bauingenieur* **30**, 3, 77–89, Mar. 1955.

Paper gives efficient approximate methods for statical calculations of modern bridge engineering. Exact mathematical treatment of a grillage of beams is very lengthy and cannot usually be applied for practical purposes. In analyzing their tests of interconnected beam systems, Morice and Little [AMR **7**, Rev. 3197] showed the excellent possibilities of the well-known approximate method due to Guyon [*Ann. Ponts Chauss.*, 1946] and generalized by Massonet [AMR **4**, Rev. 3525].

After recapitulating theoretical results of Guyon-Massonet and after giving appropriate graphs, author examines the degree of approximation reached by the method in question on numerical examples relating partly to his own tests of a no-torsion structure. Excellent agreement is obtained in all cases. But the most valuable results of the author's deductions consist in extending and modifying the method of Guyon-Massonet in such a way that is it possible to make, quickly and simply, statical computations of any torsion and no-torsion grillage. Instructive practical examples show that the exactness of the new Sattler method satisfies perfectly all requirements usual in statical calculations.

Paper is a valuable contribution to the design and practical computation of complicated bridge systems. In addition to this, author's results can doubtless be useful in other questions of engineering, e. g., in the approximate solution of problems on anisotropic plates. V. Vodička, Czechoslovakia

2907. Clough, R. W., and Pirtz, D., Earthquake resistance of rock-fill dams, *Proc. Amer. Soc. civ. Engrs.* **82,** SM2 (*J. Soil Mech. Found. Div.*), Pap. 941, 27 pp., Apr. 1956.

Paper describes an experimental investigation of the effects of earthquakes on rock-fill dams with earthen cores. The tests were performed on 1/150 scale models of two types of dams, one having an inclined core near the upstream face, and the other having a central core. The models were subjected simultaneously to water loadings and to simulated earthquakes generated by the shaking table on which the models were constructed.

The test results show that rock-fill dams inherently are very resistant to earthquakes because of their flexible structure. No catastrophic slippages occurred in the models, even when subjected to ground accelerations exceeding the acceleration of gravity. From authors' summary

2908. Shestakov, V. M., Depression curve in earth dams and causeways after decrease of reservoir level (in Russian), *Gidrotekh. Stroit.* **23**, 4, 32–36, 1954.

2909. Gliavin, Yu. V., Investigation of tension stresses in a dam core (in Russian), *Gidrotekh. Stroit.* **23**, 4, 24–25, 1954.

2910. Bochever, F. M., Hydrogeological estimation of side seepage around a dam (in Russian), *Gidrotekh. Stroit.* **23**, 6, 26–30, 1954.

2911. Nelson-Skorniakov, F. B., Filtration through homogeneous earth dam on permeable base of definite depth (in Russian), *Gidrotekh. Stroit.* **23**, 5, 28–30, 1954.

2912. Gun'ko, F. G., Energy dissipation below a dam (in Russian), *Gidrotekh. Stroit.* **22**, 9, 27–30, Sept. 1953.

2913. Denke, P. H., The matric solution of certain nonlinear problems in structural analysis, *J. aero. Sci.* **23**, 3, 231–236, Mar. 1956.

Author applied matrix formulation of the Maxwell-Mohr equations to solve a class of nonlinear indeterminate structural problems. The method is applicable to plastic or buckling problems, and others, which are characterized by the dependence of the flexibility matrix on the internal loads.

The procedure was illustrated by solving a problem involving plastic behavior of an indeterminate sweptback wing. An iterative procedure based on the Newton-Raphson method was used to solve the general nonlinear load and deflection equations, using a high-speed digital computer.

The Ramberg-Osgood equation was used to define the stress-strain curves for the various structural elements.

Convergence for the particular problem was shown to be rapid. Author makes no attempt to establish the necessary conditions for convergence in the general case, however.

M. Lemcoe, USA

Rheology (Plastic, Viscoplastic Flow)

(See also Revs. 2851, 2854, 2869, 2873, 2884, 2894, 2932, 2933, 2938)

Book—2914. Späth, W., Flow and creep of metals [Fliessen und Kriechen der Metalle], Berlin, Metall-Verlag GMBH, 1955, 160 pp. DM 13.50.

Author quotes experimental evidence that coefficient of friction rises to 100 in high vacuum between absolutely clean surfaces; places of contact melt and weld. Process presumably applies also inside of metal during slip. Oxidation or impurities (external or internal) greatly reduces weldability in both places. Regular spacing of slip lines in loaded metal bar is related to wave length of ultrasonic vibrations which are created during slip. The highly strain-hardened state of a metal is attributed to high residual (self-equilibrating) stresses which develop as certain grains slip under load, then again join the as yet unslipped grains in the load-carrying task. In contrast, a truly plastic state arises when complete cross sections begin to slip. Elastic after-effect, Bauschinger effect, creep, fatigue, and fracture are interpreted in terms of the foregoing two types of "flow" phenomena, as well as dislocations, friction phenomena.

While the reader may disagree with the author's thesis in many places, he will find the book stimulating from beginning to end. Author also discusses many nonconventional tests for evaluating stress state and metal behavior. (Physics-type tests—noting changes in metal properties under load—rather than engineering-type tests—which are usually limited to evaluation of changes in strains.) Two hundred references are given.

G. Horvay, USA

2915. Hoff, N. J., Creep buckling, *Aero. Quart.* **7**, 1, 1–20, Feb. 1956.

Author takes up problem of creep buckling of initially curved column of idealized H-section, using a new treatment combining his own original method [AMR **7**, Rev. 2444] with reviewer's theory of primary creep [AMR **8**, Rev. 1002] and adding assumptions to take account of elastic strains in a similar manner as J. Hult [AMR **9**, Rev. 740]. Treatment is very complete, taking account of strain reversal in convex side flange.

F. K. G. Odqvist, Sweden

2916. Wachtman, J. B., Jr., and Maxwell, L. H., A bend-test method of determining the stress required to cause creep in tension, *ASTM Bull.* no. 211, 38–39, Jan. 1956.

Some materials require a rather well-defined stress to cause creep in tension. A method is described which permits the use of

bending tests to determine this stress under conditions which make tension testing difficult.

From authors' summary

2917. **Kennedy, A. J., Design of a comprehensive computer for handling complex creep problems. Part I, Part II,** *Engineer, Lond.* **200,** 5188, 2–4, July 1955; **200,** 5194, 16–32, Sept. 1955.

An electric analog computer capable of handling creep problems with complex loading programs has been developed. Basic network is a simple noninductive transmission line, made up of identical sections, in which are included certain irreversible elements (rectifiers). Voltage applied is taken as analogy of stress; charge flowing into network is integrated to provide analogy of strain. For constant stress, creep strain as a function of time is assumed to have exponential form. It has been possible to simulate such properties of creep curves as the form after a stress interruption of given duration and creep recovery curves.

In its present form apparatus is capable of reproducing creep curves with any desired load program at isothermal conditions. It is believed, though, that in future work temperature changes may be reproduced by changing characteristics of network.

F. K. G. Odqvist, Sweden

2918. **Ivey, D. G., Experiments on viscoelasticity,** *Amer. J. Phys.* **24,** 5, 373–375, May 1956.

The propagation of sound waves in rubber strips is used to illustrate viscoelastic behavior. The experiments introduce time-dependent mechanical properties of solids at a level suitable for use in undergraduate laboratories.

From author's summary

2919. **Rivlin, R. S., Solution of some problems in the exact theory of visco-elasticity,** *J. rational Mech. Analysis* **5,** 1, 179–188, 1956.

Based on AMR **8,** Rev. 3656 and AMR **9,** Rev. 734, Reiner's [*Amer. J. Math.* **67,** p. 350, 1945] equation, in which stress matrix T is related to velocity strain matrix A_1 by $T = \alpha_0 I + \alpha_1 A_1 + \alpha_2 A_1^2$, is expanded by adding terms for gradient of acceleration matrix A_2, namely $\alpha_2 A_2 + \alpha_4 A_2^2 + \ldots + \alpha_8 (A_1^2 A_2^2 + A_2^2 A_1^2)$. The cases of rectilinear laminar flow, torsional flow between parallel disks, and Couette flow between coaxial cylinders, previously considered [AMR **2,** Rev. 1024], are accordingly generalized. Reference is made to AMR **6,** Rev. 3466, in which experimental measurements of torsional flow of polyisobutylene solutions are interpreted as showing that cross stresses arise from non-vanishing of α_3. However, J. E. Roberts [Proc. Second inter. Congress Rheology, London 1954] claims to have repeated the experiments with contradicting results. This discrepancy has not yet been cleared up.

This reviewer is of opinion that the question of interpretation of Weissenberg effects [*Nature* **159,** p. 310, 1947] has received theoretical treatment far in advance of scanty experimental quantitative material, and that efforts might usefully be concentrated to provide latter.

M. Reiner, Israel

2920. **O'Connor, C., Residual stresses and their influence on structural design,** *J. Instn. Engrs. Austral.* **27,** 12, 313–321, Dec. 1955.

Rolling and thermal residual stresses in structural steel of various shapes were measured with a Metzger extensiometer over eight-inch gage lengths. Results agree well with similar tests by other investigators and are a contribution to the large number of measurements necessary for adequate evaluation of the significance of residual stresses. Author discusses elasto plastic behavior of structural beams and columns under influence of such residual stresses when subjected to different loading condition. Conclusions should be of aid in refining allowable load values.

J. Frisch, USA

2921. **Hodge, P. G., Jr., Displacements in an elastic-plastic cylindrical shell,** *J. appl. Mech.* **23,** 1, 73–79, Mar. 1956.

Paper derives solution for cylindrical shell with rigid reinforcing rings at ends, under uniform external pressure which is supposed gradually increased to the collapse load. Shell is assumed to have idealized elastic and plastic properties corresponding to a sandwich-plate type of construction. General equations for elastic-plastic deformation are derived. Numerical solution is given for a particular shell geometry, carrying load up to 98% of collapse; maximum radial displacement is found to be less than five times maximum elastic displacement. An approximate solution is also obtained which avoids difficulties met with in the more exact solution in certain cases of shell geometry.

P. S. Symonds, USA

2922. **Baker, A. L. L., High building frames and foundations,** *Proc. Instn. civ. Engrs.* **4,** 2 (III), 228–245, Aug. 1955.

Paper discusses various elastic theories and simplifying assumptions which have been used by practising structural engineers.

A brief explanation of the plastic-hinge theory for reinforced-concrete frameworks is given and applied to the example of a four-bay frame any number of stories in height. Bending-moment formulas are derived for sway and vertical load moments which apply for limiting ratios of stiffness, etc., covering a large range of frameworks. The example illustrates a type of general solution which could be obtained by use of the plastic-hinge theory for single- or multi-bay building frames.

The design of raft foundations suitable for tall buildings is discussed, and some details are given of buildings in Johannesburg.

From author's summary by P. Csonka, Hungary

2923. **Greenspon, J. E., An approximation to the plastic deformation of a rectangular plate under static load with design applications,** *David W. Taylor Mod. Basin Rep.* 940, 21 pp., June 1955.

Methods are given for estimating the loads and permanent sets of rectangular panels just beyond the onset of plasticity as well as for estimating very large sets and the sets corresponding to the approximate ultimate loads; the intermediate region corresponding to moderate permanent deformations is not discussed.

From author's summary

2924. **Savitskiĭ, K. V., and Zagrebennikova, M. P., On the nature of the plastic deformation of the surface layers of rubbing bodies** (in Russian), *Dokladi Akad. Nauk SSSR (N.S.)* **103,** 4, 605–608, 1955 (Translated from Russian by M. D. Friedman, 572 California St., Newtonville 60, Mass.).

Failure, Mechanics of Solid State

(See also Revs. 2863, 2921, 2922, 3102)

2925. **Coffin, L. F., Jr., Design aspects of high-temperature fatigue with particular reference to thermal stresses,** *Trans. ASME* **78,** 3, 527–532, Apr. 1956.

A phenomenological criterion for fatigue failure caused by thermal cycling or by constant-temperature strain cycling is proposed by author. The equation derived for the purpose is of the form

$$N^k \Delta\epsilon_p = C$$

where N is the number of cycles to failure, $\Delta\epsilon_p$ the plastic-strain change, exponent k a constant, and C a constant. Application of the criterion to design problems is discussed. A brief list of further areas for investigation is appended. A pertinent bibliography covering previous work of a related nature is provided.

Reviewer considers that the experimental work used as a basis for the criterion, and which was obtained at the Knolls Atomic Power Laboratory, although not extensive, will be interesting to those working in the field of fatigue related to thermal cycling.
W. H. Hoppmann, II, USA

2926. Hebrant, F., Louis, H., Soete, W., and Vinckier, A., Study of the mechanical relaxation of residual welding stresses under static and fatigue loading (in French), *Rev. Soudure* no. 3, 15 pp., 1955.

Paper describes the development of tests and results on the effects of purposeful mechanical single overloading, followed by total unloading, of various typical simplified welded joints in structural steel, designed to induce limited and controlled plastic flow and thereby reduce or eliminate the conditions of internal residual stress caused by the welding process, particularly in redundant assemblies.

The effect is definitely obtainable, but its application in practical cases is deemed problematic as the proper distribution of the right overloads in all parts of a complex structure can hardly be produced, especially in the face of unknowns such as section properties and dimensions within or adjacent to the weld regions.

Butt joints appear untreatable, while lap and buttstrap joints do lend themselves to the operation.

Fatigue properties of the specimens tested did show improvement due to the mechanical overload stress-relief process as well as the static properties, but the improvement could only affect the strain distribution part of the notch effect and could not have any bearing on the importance of geometric notches present at the welds.
B. Posniak, USA

2927. Bowman, C. E., and Dolan, T. J., Studies of the biaxial fatigue properties of pressure-vessel steels, *Weld. J.* **34,** 1, (Weld. Res. Suppl.) 51-s-59-s, Jan. 1955.

A study was made to evaluate the effect of welding, notches, attachments, and other fabrication operations upon the resistance of pressure-vessel steels to repeated loading. Specimens of ¾-in. plate were subjected to biaxial stresses in a special loading fixture. The results are compared to previously reported studies of similar 10- × 17-in. specimens of the same steels. The earlier studies included plates of A 201 Grade A and A302 Grade B steel in the unnotched and notched conditions and plates with longitudinal submerged-arc welds. Those studies were extended to include plates with hand-deposited welds or with welded attachments that simulated nozzles. Other preliminary studies were made to determine the effect of polishing the surface of the plate and the effect of the direction of rolling relative to the direction of maximum principal stress. Similar data on the resistance to repeated loading of unnotched and notched specimens of A225 Grade B and Carilloy T-1 steels are also reported. Each fatigue curve was determined by the results from ten or more plates to show the variation in fatigue strength for a life of about 3000 to 500,000 repetitions of the pressure cycle.
From authors' summary

2928. Fisher, W. A. P., and Winkworth, W. J., The effect of tight clamping on the fatigue strength of joints, *Aero. Res. Counc. Lond. Rep. Mem.* 2873, 10 pp., 1955.

The effect of clamping on the fatigue strength of joints in aluminum alloy is investigated experimentally. Tests on Z-section stringers connected to long, slotted cleats give an endurance for tightly clamped joints about nine times that for unclamped joints. Tests of bolted joints in aluminum-alloy sheet material show still greater improvement for very tight clamping. In both series of tests, the clamping tends to cancel the weakness in fatigue of a loaded hole.

Tight clamping is considered to have many applications in design of aircraft joints from the standpoint of fatigue strength.
From authors' summary

2929. Thurston, R. C. A., and Field, J. E., Fatigue strength under bending, torsional and combined stresses of steel test pieces with stress concentrations, *Proc. Instn. mech. Engrs.* **168,** 31, 785–796, 1954.

Paper extends work by Gough, Pollard and Clenshaw [AMR **6,** Rev. 133]. Identical steels are used and fatigue specimens are pierced with diametral holes. Results can be plotted to conform to elliptic arc. Strength reduction factors are compared with those for V-notch. Authors also give results for transition fillet and splines.
R. N. Arnold, Scotland

2930. Kasatkin, B. S., Destruction mechanism of specimens subjected to standard notched-bar impact bending tests (in Russian), *Izv. Akad. Nauk SSSR Otd. tekh. Nauk* no. 7, 75–83, July 1955.

Breaking standard specimens in a special jig (shown) which permits varying the extent of their bending and examining them under a microscope showed that plastic deformation at the bottom of the notch is limited to an area of an equilateral triangle, the base of which corresponds to the bottom of the notch. Maximum stresses are located next to base and totally disappear at the apex of the triangle 1.5 to 2 mm below the bottom. Microcracks start near the base and propagate as fissures along the sides of the triangle. In ductile metals, two major cracks are formed simultaneously, both beginning at the bottom of the notch, but only one develops further leading to a break. In brittle metals, the break is preceded by deformation of a very narrow and thin surface layer, and lower temperature further narrows the zone of plastic deformation. Only little energy is spent in plastic deformation of brittle metals before the crack starts and concentrates the stresses at its end.
J. D. Gat, USA

2931. Hu, L. W., An experimental study on the fracture of metals under hydrostatic pressure, *J. Mech. Phys. Solids* **4,** 2, 96–103, Feb. 1956.

Experimental investigation on the plastic behavior of an aluminum alloy Alcoa 61S-T6 subjected to triaxial stresses was conducted. It was found that fracture strength was not affected by hydrostatic pressure, but ductility decreased as hydrostatic pressure increased.
From author's summary

Material Test Techniques

(See also Revs. 2887, 2889, 2916, 2929, 2930, 2948)

2932. Green, A. P., and Hundy, B. B., Initial plastic yielding in notch bend tests, *J. Mech. Phys. Solids* **4,** 2, 128–144, Feb. 1956.

Theoretical plane-strain solutions are presented for the initial plastic yielding of notched bars bent either under four-point loading or as in the Izod or Charpy tests, and having either a V-notch or a notch whose root is a circular arc. These solutions provide estimates of the stresses in the plastic regions, the mode of yielding, and the corresponding yield-point loads. Calculations are made of the maximum tensile stress below the notch root and of the smallest depth of notch for which initial yielding does not extend to the notched surface. The theoretical solutions are confirmed by experimental observations of the yield-point loads in slow bend tests, and of the modes of deformation in slow and impact tests; the latter are found to be very similar. The ductility

transition is discussed in relation to this and other work.

Reviewer believes this to be a valuable contribution to a proper understanding of notch-bend tests. A precise appreciation of the significance of these tests will be possible only with a fuller understanding of fracture conditions.

J. F. W. Bishop, Scotland

2933. Jones, M. H., and Brown, W. F., Jr., An axial loading creep machine, *ASTM Bull.* no. 211, 53–60, Jan. 1956.

Factors influencing the eccentricity of loading in tension testing, and a creep machine designed to reduce bending stresses in tension creep to a minimum are discussed.

From authors' summary

2934. György, L., and Nádor, B., Determination of Young's modulus for glass (in Hungarian), *Mérés és Automatika* **3,** 11, 349–353, Nov. 1955.

Article gives a short summary on the elasticity of glass and describes measurements and methods thus far applied in the determination of its Young's modulus. The authors' own investigations are then explained, and two methods, the statical and the acoustic, for the determination of Young's modulus are described. By perfecting the methods thus far applied abroad, the two procedures have been made suitable for determining the precision of measurement, that is, the error of measurement. Using the statical method, Young's modulus was determined from the deflection of the glass bar loaded at the middle with a precision of ±150 kg/mm², so that the relative error of measurements is 2.2%. The precision attained in the measurement by the acoustic method is about 140 kg/mm²; the error is thus 2%. By combining the two methods, the relative error of measurement can, in certain cases, be reduced to about 1%. From authors' summary by I. Korányi, Hungary

2935. Braun, A., Significance of dependence of Vickers hardness upon load (in German), *Z. Metallk.* **46,** 7, 499–503, July 1955.

Author considers two limiting values of Vickers hardness, one of which is approached when the impression is of extremely small dimensions, and the other with very large impressions. The transition between these two values is expressed by algebraic equations which account for such factors as work-hardening and increase of heterogeneity of the crystal structure of the metal and elastic recovery of the indentation. Author discusses light contrast at the indentation boundary and optical methods for determining indentation dimensions more accurately.

For test data and certain details, the two references given should be consulted. E. B. Shand, USA

2936. Maloney, W. J., Jr., The application of nondestructive testing by a manufacturer of military aircraft, *Aero. Engng. Rev.* **15,** 4, 76–84, Apr. 1956.

Mechanical Properties of Specific Materials

(See also Revs. 2851, 2900, 2927, 2931, 2934, 2948, 2950, 2952, 3063, 3073)

Book—**2937. Dawihl, W., and Dinglinger, E., Handbook of carbide tools [Handbuch der Hartmetallwerkzeuge. Vol. II, Rundlaufende Werkzeuge, spanlose Formgebung, Bergbau-und Diamantwerkzeuge],** Berlin, Springer-Verlag, 1956, xii + 436 pp. DM 34.50.

Book contains eighteen chapters which are more or less independent and cover the field of carbide application from a variety of viewpoints. Carbides and their application in industrial tooling for machine-shop production, mining, oil-well drilling, and hand tools are discussed. Basic properties of sintered carbides in relation to microstructure, temperature, and vibrations are illustrated. Design of carbide tools for deep-hole drilling in metals as well as for boring in stone, glass, ceramics, plastics, and wood is included as is also the application of carbides to broaching and reaming. German standards, terminology, and design features of carbide milling cutters for different applications are listed: Composition and design of carbide tools for drawing wire and tubing and assembly and mounting of dies for deep drawing and forming; heat resistant parts made of tungsten carbide for use in machinery where high-temperature service is a major factor. Uses of diamonds in hardness testing, in grinding carbide tools, and as tool tips are treated in separate chapters. Bonding of diamond particles in grinding wheels, mounting of diamond tool points, and reclamation of diamond particles are covered. Hardness tests on ferrous and nonferrous materials with metallic and diamond indentors are compared. Boron carbide, its production and sintering, mechanical and chemical properties, and applications, are treated.

A. O. Schmidt, USA

2938. Mathauser, E. E., and Deveikis, W. D., Investigation of the compressive strength and creep lifetime of 2024-T aluminum-alloy skin-stringer panels at elevated temperatures, *NACA TN* 3647, 29 pp., May 1956.

The experimental results of an investigation to determine compressive strength and creep lifetime of 2024-T (formerly 24S-T) aluminum-alloy skin-stringer panels at room temperature and at 400 F are presented. The results of strength tests at room temperature and 400 F are compared with predicted strength obtained from methods given in the literature for estimating crippling strength of short panels and for predicting column strength of longer panels. Creep lifetime curves are presented for four values of slenderness ratio, and creep characteristics of the panels are discussed. A method which makes use of isochronous compressive stress-strain curves for predicting creep lifetime of panels is presented. From authors' summary

2939. Jensen, J. W., and Rowland, J. A., Jr., Manganese-copper high-damping alloys, *Prod. Engng.* **27,** 5, 135–137, May 1956.

Mechanical properties, fabrication characteristics, and potential applications of alloys having the unusual combination of high tensile strength and high damping capacity are discussed.

From authors' summary

2940. Loewen, E. G., Thermal properties of titanium alloys and selected tool materials, *Trans. ASME* **78,** 3, 667–670, Apr. 1956.

Thermal conductivity has been determined of five titanium alloys, four high-speed steel alloys, and four types of tungsten-carbide tool materials, over a temperature range of 200 to 1000 F. In addition, the specific heat of two titanium alloys has been found between 70 and 1400 F. From author's summary

2941. Bopp, C. D., and Sisman, O., Stress-strain curves for reactor-irradiated plastics, *Nucleonics* **14,** 3, 52–53 (Data Sheet 12), Mar. 1956.

2942. Fahy, F., A review of plastics for use in the aircraft industry, *Coll. Aero. Cranfield Note* 43, 19 pp., Mar. 1956.

2943. Hulbert, G. C., Plastic structures, *J. roy. aero. Soc.* **60,** 542, 114–120, Feb. 1956.

2944. Hartman, A., The mechanical properties of glass-cloth reinforced plastics at room temperature (in Dutch), *Nat. Lucht-Lab. Amsterdam Rap.* M. 1991, 17 pp. + 10 tables, 47 figs., Aug. 1955.

The results of an investigation to determine the mechanical properties of materials made from asbestos or glass fibers and plastics in relation to their use in aeronautical constructions are

reported. The mechanical properties determined are strength and stiffness in extension, bending, and compression tests. The influence of the characteristics of glass content, type of weave, type of finish, type of resin, and cross and parallel laminating is examined not only for the materials in dry state but also for the materials in wet state after immersion for 3 hr in boiling water. The value of this accelerated test was checked by comparison with immersion tests of long duration at room temperature.

The influence of these various factors on the afore-mentioned properties of the materials is discussed and the results are reproduced in tables and diagrams.

D. DeMeulemeester, Belgium

2945. Niven, C. D., On the effect of speed on the kinetic friction of some plastic materials on ice, *Canad. J. Phys.* **34,** 4, 362-369, Apr. 1956.

Curves are shown illustrating the effect of speed on the friction of teflon, bakelite, nylon, terylene, plexiglas (perspex), cellulose acetate, polystyrene, and polyethylene, sliding on ice at −5C and −15C. Bakelite and teflon had the lowest kinetic friction of the materials examined. Terylene also slid easily, but polyethylene had rather high friction. Departure from Amontons' law was very evident at high speed. From author's summary

2946. McCallion, H., and Davies, D. M., Behaviour of rubber in compression under dynamic conditions, Instn. mech. Engrs. Preprint, 10 pp.

The methods and results are described by which the variation of stiffness, hysteresis, and resilience of a natural rubber component under cyclic sinusoidal loading is measured as a function of frequency (1.33–1200 cpm), amplitude of deformation (0–10%) of free length, and temperature (−60C to 125C). A tentative expression is developed to agree with the observed data.

R. G. Boiten, Holland

2947. Stone, W. K., Schiefer, H. F., and Fox, G., Stress-strain relationships in yarns subjected to rapid impact loading: I. Equipment, testing procedure and typical results; II. Breaking velocities, strain energies, and theory neglecting wave propagation, *J. Res. nat. Bur. Stands.* **54,** 5, 269–280, May 1955.

In part I, impact apparatus for elongating yarns at rates of straining up to 500,000% per minute is described and the procedure for obtaining load elongation curves is discussed. These rates of straining are, however, still considerably lower than the critical velocities of rupture of most textile materials. The principle of the method is to record cinematographically the positions of the two ends of the yarn while it is being stretched, and the load-time curves are obtained by a double differentiation of the displacement-time curves. Part II discusses the theory of the method and gives calculated values for critical breaking velocities of cotton, nylon, and undrawn nylon; these are 130, 228, and 550 mps, respectively. The practical application of this work to safety lines is discussed.

H. Kolsky, USA

Mechanics of Forming and Cutting

(See also Revs. 2898, 2937, 2940)

2948. Peterson, R. E., Mochel, N. L., Conrad, J. D., and Gunther, D. W., Large rotor forgings for turbines and generators, ASME Ann. Meet., Chicago, Ill., Nov. 1955. Pap. 55-A-215, 31 pp. + 27 figs.

Authors present an excellent, although necessarily brief, review of present situation regarding design and manufacture of large rotors. Size of units has increased until present practice reaches such figures as 108-in. ingots weighing 4–500,000 lbs. Development of the smaller 3600-rpm rotors is shown to have advantages over larger 1800-rpm rotors in that smaller forgings are needed, and that the smaller forgings can be acceptance-tested with less ambiguous results. The failure of a large rotor at Ridgeland Station (Chicago) is taken as an indication of such results; authors then discuss the advantage of a design approach for large 1800-rpm rotors wherein multiple rings are shrunk on a central shaft.

Paper treats *Materials:* Ni-Cr-Mo-V and Cr-Mo-V steels of 100,000 psi Y.S., 120,000 psi T.S.; 16% Elong. and 40% R. of A. Austenitic and 12% Cr steels are shortly expected to be used in high-temperature service. *Manufacture:* Emphasizes need for excellent relations between turbine and steel manufacturers. *Machinery:* They do as little machining as possible after final stress-relieving. *Testing and Acceptance:* Chemical analyses are turning to lower P and S. Effects of H are being intensively investigated by an ASTM group. *Mechanical Properties:* Tension tests in longitudinal, tangential, and radial directions; notch impact, notch bend, and transition temperature tests are all used. *Sulphur prints* are still used to check ingot corner segregation. *Bore inspection* guards against major ruptures, fissures, and, with modern equipment, against very small defects. Rotors are examined all over by *Magnaflux. Ultrasonic* examination at 2¼ mc is made over entire surface, any indication of discontinuity being further explored with 1 mc; work is done by engineers, not inspectors. Much emphasis on this work; major concern is with location and positive identification of defect. Final disposition of rotor is made by metallurgists and mechanical engineers after discussing results of ultrasonic and tensile tests. Possible causes of *rotor instability* (transient or permanent bowing) are: nonuniform creep, incomplete metallurgical transformation, relaxation of stresses by machining, relaxation of nonuniform stresses by caulking of blades.

Rotor design criteria are discussed in terms of overspeeding (or, synonomously, overstressing) with subsequent redistribution of stress, and in terms of transition temperature; need for adequate ductility is emphasized. Fatigue from bending, misalignment, and stopping-and-starting is considered at some length; emphasis primarily is on shape and orientation of defects, their size and number being less important.

Tests of model rotors gave results in agreement with a theory of the mode and sequence of failure at Ridgeland, failure occurring when mean stress equaled tensile stress.

C. C. Osgood, USA

2949. Lueg, W., and Treptow, K.-H., New investigations on the drawing and "pushing-in" of steel rods. Part I, Part 2, (in German), *Stahl u. Eisen* **75,** 3, 162–169, Feb. 1955; **75,** 12, 769–776, June 1955.

Part I describes a comparative experimental and theoretical study of the forces required in (a) pushing round bars into, and (b) pulling them through a drawing die. Authors show that, for a constant percentage reduction of cross section, the force for pushing (extrusion) is higher than for normal drawing; furthermore, that the specific drawing (extrusion) force decreases gradually with increasing bar cross section and approaches an asymptotic value.

In part II, the elastic distortion (diametrical expansion or contraction) of the bar as it leaves the die was studied. This elastic strain was generally positive, but for any specific die angle it varied with the length of the cylindrical die extension, its value invariably reaching a minimum at a certain length of said extension. The minimum was either slightly positive or negative, depending on the die angle.

A substantial portion of part II represents an attempt to explain, on a rational basis, the asymptotic fall of the drawing stress mentioned earlier. This was ascribed to nonpenetrating deformation of

the thicker bars, although the theoretical speculations and experimental results brought in support of this contention cannot be regarded as convincing.

As a whole, these two papers can be recommended as informative and interesting contributions to the subject.

N. H. Polakowski, USA

2950. **Yazawa, S., On the plug-drawing of steel tubes,** *J. mech. Lab., Tokyo* **1,** 2, 55–56, 1955.

2951. **Vilium, B. F., Hydrodynamic theory of horizontal centrifugal casting** (in Russian), *Izv. Akad. Nauk SSSR Otd. tekh. Nauk* no. 10, 39–46, Oct. 1954.

When smelted metal is poured into hollow molds, it rotates around the horizontal axis of symmetry of the cylinder and, due to its viscosity, is absorbed in the cylinder walls. At sufficiently large values of rotation, the fluid arranges itself in layers on the interior surface of the cylinder. On this principle is founded the method for obtaining castings with cylindrical cavities; it is known as centrifugal casting. The theoretical investigation of the process of centrifugal casting poses an additional problem, because it is necessary to consider the large number of factors which influence the formation of the casting.

Existing theories of centrifugal casting are founded on the consideration of an extremely simplified system of phenomena, where the smelt is considered as an ideal fluid. Such a system will, in no sense, correlate reality; in the same sense, an ideal fluid does not adhere to anything and, finally, cannot be involved in the rotatory motion of the cylinder. Because of this, in the same elementary theoretical system, smelt must be considered as a viscous fluid. The essential questions in the theory of centrifugal casting, pertaining to the form of the free surface of the viscous metal, regardless of the angular velocity in the casting mold, require the differential between the maximum and minimum wall thickness of the casting and show themselves to be questions in the hydromechanics of viscous fluids.

The mathematical determination of the hydromechanical problem of a viscous fluid poses some very great difficulties, such that it becomes necessary to consider a complicated system of nonlinear differential equations with peculiar derivatives. Precise integration of these equations is rarely successful. In the present paper, consideration is given to the problem of the motion of a heavy viscous fluid rotating around the horizontal axis of symmetry of a cylinder. The problem is solved by means of approximate integration of the differential equations of the fluid for the case when the thickness of the fluid layer is small in comparison to the radius of the cylinder.

N. M. Matusewicz, USA

2952. **Suddarth, S. K., and Angleton, H. D., Nail popping, a result of wood shrinkage,** *Purdue Univ. agricul. Exp. Sta. Bull.* 633, 41 pp., Jan. 1956.

The investigating wood technologists presented and erroneously "proved" their hypothesis that nail-shank surfacing or deformation has no significant influence on amount of observed nail popping on the basis of fallacious test procedures which resulted in test data not representative of actual field conditions.

Thus, the nails were driven directly—not, for example, through gypsum-board—and fully into the lumber. Hence, they penetrated it with their full lengths. None of the fasteners, especially not the threaded nails included in this study, were designed for such a purpose.

The nailed test samples were conditioned from "green" to oven-dry to "green" status. It is misleading to base recommendations for average field conditions on test data secured by drying lumber to zero per cent moisture content when the average moisture content of framing lumber ranges from 7 to 12% in the dry southwestern states and from 9 to 14% in the remainder of this country.

Authors uphold their hypothesis in contradiction to their own test data and finally conclude that surface condition and nail-shank deformation showed "slight influence" on amount of observed pop, despite the fact that (1) the 10% longer, cement-coated, plain-shank nail popped 50% more than the bright threaded nail, (2) a statistical analysis of the test data shows "nail type to be strongly influential" on the test data, "significant beyond the 1% level," and (3) popping shown by the plain-shank nail during cyclic exposure tests amounted to 5.7 and 3.9 times that of the threaded nail (in agreement with test data obtained elsewhere).

The described tests confirm that nail diameter has little bearing on amount of nail popping and that a direct relationship can be assumed between length of nails and amount of popping. In light of the latter observation, the gypsum industry adopted—as long as 1 1/2 years ago—specially designed, relatively short, properly threaded nails for fastening gypsumboard, since these nails provide satisfactory holding power and reduce nail popping to a minimum.

E. G. Stern, USA

2953. **Mierzejewski, J., Speed of measurement with pneumatic length measuring device** (in German), *Feingerätetech.* **4,** 12, 558–562, Dec. 1955.

Author analyzes the response to a step input for back-pressure-type pneumatic comparator. Experimental confirmation is good in the slow region (10 to 160 sec), but is not checked in the more important high-speed region. That response speed is inversely proportional to the volume of the measuring passages, and proportional to the size of the initial restriction is well known. More controversial is the conclusion that response is slowed down by increase in supply pressure. There is no experimental confirmation of this last statement.

E. G. Loewen, USA

2954. **Linnik, Yu. V., and Khusu, A. P., Mathematical statistical account of surface-unevenness contours following polishing** (in Russian), *Inzhener. Sbornik, Akad. Nauk SSSR* no. 20, 154–159, 1954.

A curve of the surface profile can be assumed as a result of accidental process, representing heights of unevennesses on selected axis of abscissae. A parameter is introduced into a standard deviation, which accounts for speed of damping the correlation ratios. This enables answering several questions important for characteristics of friction surfaces. These questions are discussed: (1) Probability of the total length of particles emerging above the mean line of profile; (2) probability of the total sum of areas of crests over the horizontal surface.

From authors' summary by S. Kolupaila, USA

Hydraulics; Cavitation; Transport

(See also Revs. 2815, 2912, 2951, 2994, 2997, 3039, 3042, 3133, 3146)

2955. **Ippen, A. T., Hydrodynamics—a review of von Karman's contributions,** *J. aero. Sci.* **23,** 5, 438–443, 499, May 1956.

This birthday tribute reviews the significant developments in five divisions of hydrodynamics in which von Karman's contributions are essential. The paper's importance is that it gives a general picture showing von Karman's role and influence, rather than a restricted discussion of his papers. The subject matters are: Boundary layer and turbulence, von Karman vortex trails, impact of seaplane floats on water, hydraulic analogy to supersonic motion, and hydraulic machinery.

Von Karman's far-reaching direct and indirect influences are clearly brought out and documented by 25 references. Many of his ideas are mentioned in the paper which still await detailed research. His "characteristic approximate manner", applied

successfully to a great variety of problems by him and by workers in the field, is described in this strongly recommended paper.

V. G. Szebehely, USA

Book—2956. Addison, H., Land, water and food, London, Chapman & Hall, Ltd., 1955, xii + 248 pp.

Author, for 30 years professor at the Fouad University at Giza, Egypt, and well-known for his valuable books "A treatise on applied hydraulics" and "Hydraulic measurements," gives "a topical commentary on the past, present, and future of irrigation, land reclamation, and the food supplies they yield". Book is a brilliant introduction to hydrology and hydraulic engineering, based on author's personal experience and covering vital hydraulic problems of Egypt, Levant states, India, Australia, America, and England. A list of selected sources is added. Book is very interesting for civil engineers and particularly for students.

S. Kolupaila, USA

2957. Acosta, A. J., A note on partial cavitation of flat plate hydrofoils, *Cal. Inst. Technol. Hydrodyn. Lab. Rep.* E-19.9, 7 pp. + 4 figs., Oct. 1955.

Recently, Tulin and Wu have treated the problem of fully developed cavitation on flat plate and cambered foils. In these treatments, the length of the cavity is always greater than the chord of the hydrofoil and the cavity is assumed to start at the leading edge of the plate. The purpose of this note is to extend Tulin's work to account for partial cavitation, i.e., when the cavitation bubble is less than the hydrofoil chord.

From author's summary by H. H. Anderson, Scotland

2958. Medici, M., Fluid dynamical analogy of cavitation phenomena and the sound barrier (in Italian), *Ingegnere* **29**, 745-750, July 1955.

Author remarks in the introduction that when the blades of a turbo-air compressor reach a speed close to that of sound, a decrease in output occurs similar to the phenomenon which takes place in hydraulic machines during cavitation. In two successive paragraphs, he describes and discusses the two phenomena, cavitation in the turning blades of hydraulic machines and compression shock in the turbo-air compressors, respectively, also the variables upon which they depend and some characteristic parameters.

Some information furnished by the author on theoretical and technical approaches to be followed in order to avoid cavitation phenomena in centrifugal pumps and the shock phenomena on reaching the speed of sound in turbo-air compressors is of particular interest. Reviewer believes that a more direct comparison between the two phenomena would have been desirable.

A. Russo Spena, Italy

2959. Pode, L., The diffusion of air into a pulsating cavitation bubble, *David W. Taylor Mod. Basin Rep.* 804, 17 pp., Mar. 1955.

A calculation is made of the growth of a cavitation bubble through the diffusion of air induced by the spherically symmetrical mechanical motion of the pulsating bubble. The equation of motion of the boundary is derived from energy considerations. Kinetic theory is applied to determine the boundary conditions for the diffusion equation. The diffusion equation is treated by means of the approximation that the diffusion takes place in a very small layer of water about the boundary. Numerical results are obtained for one case of a strongly pulsing bubble. It is found that the growth per period is relatively small unless the bubble is extremely tiny.

From author's summary by S. H. Crandall, USA

2960. Wadlin, K. L., Shuford, C. L. Jr., and McGehee, J. R., A theoretical and experimental investigation of the lift and drag characteristics of hydrofoils at subcritical and supercritical speeds, *NACA Rep.* 1232, 22 pp., 1955.

A method, tested by experiments, has been developed to calculate lift and drag of hydrofoils from aerodynamic data, below sub cavitation levels. While the theory predicts an abrupt increase of drag at the critical speed, experiments show that it is gradual. In neither case of submergence tested was there any effect of the limiting speed of wave propagation on either the lift curve slope or the angle of zero lift of the two model hydrofoils.

Y. V. G. Acharya, India

2961. Makeiev, G. S., Hydraulic method of preventing cavitation damage in turbines (in Russian), *Gidrotekh. Stroit.* **23**, 8, 30-32, 1954.

2962. Paderi, F., On the ascending segment of the profile for spillway crests (in Italian), *Pubbl. Fac. Ing. Univ. Pisa* (15) no. 738, 16 pp., 1955.

After a study of Bazin experimental data on weir-nappe profiles, author recommends a quarter ellipse for the ascending part of the profile. Reviewer believes that a better approach is to be found in chap. VIII, sec. C, of "Engineering hydraulics", edited by H. Rouse.

E. O. Macagno, Argentina

2963. Smith, A. G., and Allen, J. E., Water and air performance of seaplane hulls as affected by fairing and fineness ratio, *Aero. Res. Counc. Lond. Rep. Mem.* 2896, 27 pp., 1954.

An analysis is made of data on the variation of hull air drag with length/beam ratio and degree of local fairing, and of the maximum beam loading permitted for reasonable hydrodynamic performance. The effect of length/beam ratio on hull structure weight is also very briefly discussed.

Considerable reduction of surface-area drag coefficient is shown to be possible by extending the length/beam ratio above that in normal use, keeping the height of waterborne load constant. The length/beam ratio can usefully be defined in terms of the forebody-length/beam ratio. If this be increased from 3.5 to 7, the surface drag coefficient with an unfaired two-step hull decreases from 1.6 to 1.35 times that of a body of revolution of the same length and maximum cross-sectional area. With a transverse step faired with a 10 : 1 straight fairing in elevation, the reduction is from the order of 1.15 to 1.10 and with a step faired in planform and elevation from the order of 1.20 to 1.15. The usefulness of the last step form is high because no applied ventilation is required to make it operationally acceptable. Also its drag could be further reduced if applied ventilation were permitted.

A similar order of total drag reduction is also possible if the forebody-length/beam ratio be increased so as to keep constant the product of the beam and the square of the forebody length. Under these conditions there is negligible increase of over-all surface area, and the water performance for a given water load and hull height is reasonably constant as beam loading is increased from the order of $C_{\Delta_0} = 0.9$ at a forebody-length/beam = 3.5 to $C_{\Delta_0} = 3.7$ at 7.0. The limit to which the length/beam ratio can be increased will probably be determined by the minimum hull volume or width required for stowage purposes since this decreases fairly quickly, or seating width in passenger aircraft.

Hull weight is probably decreased a little with increase of length/beam ratio when the product of the beam and the square of the forebody length is kept constant.

Charts are given to help in the selection of the best hull form for any given design conditions. From these may be estimated dimensions, air drag, and the maximum beam loading to give a reasonable standard of water performance.

From authors' summary

2964. Parkin, B. R., Perry, B., and Wu, T. Y.-T., Pressure distribution on a hydrofoil running near the water surface, *J. appl. Phys.* **27**, 3, 232-240, Mar. 1956.

The effect of the free surface on the pressure distribution on the

upper side of a shallow-running hydrofoil is considered from a general point of view. Previous theoretical and experimental work is reviewed in order to compare the range of flow variables for which each treatment of the surface proximity problem is valid. A qualitative theoretical expression for the pressure is developed. This result shows the relative importance of the pertinent parameters, and it is shown to agree qualitatively with previous experiments as well as with new pressure measurements made in the free-surface water tunnel. The foregoing considerations reinforce the view generally held in the past that the principles of potential theory when properly applied to hydrofoils, even at shallow submergences, may be expected to lead to valid and useful results.

From authors' summary by M. S. Macovsky, USA

Incompressible Flow: Laminar; Viscous

(See also Revs. 2910, 2911, 2955, 2957, 2960, 2963, 2964, 2994, 2997, 3011, 3012, 3013, 3014, 3023, 3031, 3033, 3039, 3040, 3041, 3049, 3060, 3078, 3090, 3100, 3125, 3127, 3128)

2965. Curle, N., Unsteady two-dimensional flows with free boundaries. I. General theory, *Proc. roy. Soc. Lond. (A)* **235,** 1202, 375–381, May 1956.

Paper considers the initial development of steady two-dimensional flows. The motion of an incompressible inviscid fluid with free boundaries is considered by transforming into the hodograph plane of the steady flow. The equation of the free boundary and the velocity potential are expanded in powers of $e^{-\lambda t}$. Thus

$$\phi(x,y,t) = \phi_0(x,y) + e^{-\lambda t}\phi_1(x,y) + \ldots .$$

where ϕ_0 is known steady-state solution and ϕ_1 is to be determined. The exact boundary condition, which is the unsteady form of Bernoulli's equation, is applied on the free boundary which is not taken as a streamline.

From author's summary by S. I. Pai, USA

2966. Curle, N., Unsteady two-dimensional flows with free boundaries II. The incompressible inviscid jet, *Proc. roy. Soc. Lond. (A)* **235,** 1202, 382–395, May 1956.

Paper applies the general theory of part I [see preceding review] to the flow when a slit of finite width is suddenly opened in a large container and a jet of fluid issues into free space. The theory shows a kink at the nose of the jet which is due to mathematical defect. Author shows how to estimate a realistic shape of the nose of the jet.

From the author's summary by S. I Pai, USA

2967. Lighthill, M. J., The image system of a vortex element in a rigid sphere, *Proc. Camb. phil. Soc.* **52,** 2, 317–321, Mar. 1956.

It is shown that a vortex element has the following image system in a rigid sphere immersed in incompressible fluid: (1) A transverse vortex element at the inverse point, equal in strength to minus the transverse resultant of the given vortex element, divided by the ratio of its distance from the center to the radius of the sphere; (2) a radial vortex element at the inverse point, equal in strength to plus the radial resultant of the given vortex element, divided by the same ratio; (3) a uniform line vortex stretching from the inverse point to the center of the sphere, equal and opposite in total strength to the radial vortex element at the inverse point.

The result opens up the possibility of computing the secondary flow produced by a sphere in a sheared stream.

H. Mirels, USA

2968. Bugliarello, G., The resistance to accelerated motion of spheres in water (in Italian). *Ric. sci.* **26,** 2, 437–461, Feb. 1956.

In experiments with spheres of steel, copper, and cement, the resistance, velocity V, and acceleration A were determined. Assuming resistance represented by the well-known term $C(\frac{1}{2}\rho V^2 S)$, the coefficient C appears as a function of $Re = VD/\nu$ and of AD/V^2. Experiments covered Re from about 10^4 to 10^5 and AD/V^2 from 0.005 to 1.0. No correlation was found for the added mass coefficient k, that appears of small usefulness.

E. O. Macagno, Argentina

2969. Kanwal, R. P., Generalized periodic singular points with applications to flow problems, *Bull. Calcutta math. Soc.* **47,** 1, 9–22, Mar. 1955.

Paper presents solution of unsteady Stokes equations for slow viscous oscillating flow past a sphere circular cylinder and elliptic cylinder in the presence of an arbitrary oscillating body force. Formulas for drag coefficient are given but no examples are shown.

L. Trilling, USA

2970. Borden, A., Wall corrections for flow about two- and three-dimensional symmetrical bodies in rectangular channels of infinite and finite lengths, *David W. Taylor Mod. Basin Rep. 864,* 34 pp., Dec. 1954.

Methods for obtaining the wall interference in the flow about two- and three-dimensional bodies at any point in a rectangular channel of infinite and finite lengths have been developed. Exact expressions have been obtained for the Rankine oval and the circular cylinder, and reasonably accurate approximations have been derived for the Rankine ovoid and the sphere. Methods for extending the results to bodies of other shapes are discussed.

From author's summary by S.-M. Yen, USA

2971. Murai, H., Theory of cascade flow for arbitrary profiles with high camber and thickness (in German), *ZAMM* **35,** 1/2, 48–54, Jan./Feb. 1955.

Paper deals with the theoretical method for clarifying the characteristics of the cascade of high solidity composed of air foils with large camber and thickness with sufficient accuracy. By a numerical example it is proved that the results obtained by the method agree well with the experimental values obtained by F. Numachi and T. Kurokawa. From author's summary

2972. Verschaffelt, J. E., Centrifugation of mixed fluids (in French), *Acad. roy. Belgique, Bull. Cl. Sci.* (5) **41,** 7, 709–717, July 1955.

Author studies centrifugation of a liquid rotating along an axis. At first, he develops the equation of the movement of a perfect fluid considering a fixed system of axis and another rotating along the fixed axis. He arrives that $\Gamma = G + V = constant$ [1], where Γ is a generalized thermic potential, G is the thermic potential by unity of mass of liquid, and V is the potential in the field of the centrifugal force.

After some considerations regarding incompressible liquids, author studies the movement of a mixed fluid where the density is a function of the distance to the axis of rotation. The condition of equilibrium for each constituent is always the Eq. [1]. When the liquid is not in equilibrium, the general theory of diffusion in an intense gravitational camp is applicable.

For a fixed observer he considers the movement as the composition of circular and radial ones.

R. R. Hertig, Argentina

2973. Goodman, T. R., Linearized theory of water drop impingement, *J. aero. Sci.* **23,** 4, 399–400 (Readers' Forum), Apr. 1956.

Formulas for the trajectories of particles in the vicinity of a thin airfoil are derived. The analysis is applied to a flat plate at an angle of attack, and the efficiency of collection of particles on the surface is calculated.

The analysis assumes the flow field in the vicinity of the airfoil to be given by first-order linearized theory. To the same order of

linearized approximation, the motion of the particles relative to this flow field is governed by Stokes' law. Although this second approximation may be mathematically consistent with the first, this reviewer questions whether it makes as good physical sense.

L. Talbot, USA

2974. Pris, M. R., Static aspirators; directed natural ventilation, Part I and II (in French), *Ann. Trav. publics Belg.* **9,** 98, 160–180, Feb. 1956.

2975. Kolupaila, S., Methods of determination of the kinetic energy factor, *Port Engineer, Calcutta* **5,** 1, 12–18, 1956.

Article deals with a Coriolis' coefficient for a correct value of velocity head in the fundamental Bernoulli equation; this is a correction due to different velocities across a section. Calculation methods are shown for a circular pipe, for both laminar and turbulent flow, and for an open channel. Two methods of graphical integration are given for a circular pipe and three methods for an open channel. Complete examples are joined in condensed form. Author suggests approximative standard values for various flow conditions.

From author's summary

Compressible Flow, Gas Dynamics

(See also Revs. 2831, 2958, 3003, 3007, 3008, 3009, 3023, 3026, 3032, 3047, 3048, 3050, 3056, 3057, 3058, 3060, 3061, 3074, 3076, 3118)

2976. Warren, C. H. E., Recent advances in the knowledge of transonic air flow, *J. roy. aero. Soc.* **60,** 544, 241–252, Apr. 1956.

Paper summarizes some of the recent work on the nonlinear transonic theoretical problem, namely in two dimensions. Author gives a clear and comprehensive account of the various similarity laws and an analysis of the experimental data on the distance of the bow shock wave from the leading edge. The cross coupling between the effects of thickness and of incidence, camber, and twist is expressed in general conventional form. Author examines the validity of extrapolating the linearized equation to the limit when the main stream Mach number is equal to unity. He gives a full description of the flow patterns of the transonic region, following the work of Gullstrand, and then studies those features of the transonic air flow which are of importance in aircraft design. The violent changes of lift in aerodynamic center position which are associated with sonic line and shock-wave movements are described briefly.

G. Temple, England

2977. Takami, H., A study of the compressible flow past Tomotika-Tamada's profiles, *J. phys. Soc. Japan* **11,** 4, 446–451, Apr. 1956.

Author compares the exact solutions for the two-dimensional high subsonic flow past a certain family of profiles with results obtained by using various approximate methods. By investigating velocity distributions over the profiles, certain discrepancies are shown to exist between the exact and approximate solutions. These discrepancies are due to the difference between the characteristics of adiabatic gas and the hypothetical gas assumed by Tomotika and Tamada.

G. Power, England

2978. Matunobu, Y., Application of Imai's transonic approximation method to the compressible flow past a Kaplan bump, *J. phys. Soc. Japan* **11,** 4, 452–457, Apr. 1956.

Author applies Imai's transonic thin-wing-expansion method to the compressible flow past a Kaplan bump (calculation of the velocity potential and the velocity distribution). The results, including the fourth approximation, are compared with those obtained by means of the complete thin-wing-expansion, and with those deduced from von Karman's transonic similarity theory. Imai's method is shown to yield excellent results in comparison with von Karman's theory.

G. Sestini, Italy

2979. Willis, J. H., and Randall, D. G., The theoretical wave drag of open-nose axisymmetrical forebodies with varying fineness ratio, area ratio and nose angle, *Aero. Res. Counc. Lond. curr. Pap.* 245, 13 pp. + 5 figs., 1956.

Theoretical results due to L. E. Fraenkel, *Aero. Res. Counc. Rep. Mem.* 2842, May 1951, are applied to determine the drag force on open-nosed cylinder-type forebodies and afterbodies, with profiles of a general parabolic form (including straight line profiles), at zero incidence to a supersonic stream. The drag force is shown to be a function of parameters defining the profile geometry, these parameters being restricted in value due to use of linearized supersonic theory. Curves are drawn indicating variation of drag with profile parameters.

E. E. Jones, England

2980. Yoshihara, H., On the flow over a wedge in the upper transonic region, *Proc. second U. S. nat. Congr. appl. Mech.*, June 1954; Amer. Soc. mech. Engrs., 1955, 643–649.

2981. Stewart, H. J., A review of source superposition and conical flow methods in supersonic wing theory, *J. aero. Sci.* **23,** 5, 507–516, May 1956.

2982. Linnell, R. D., and Bailey, J. Z., Interpolation formulas for supersonic-hypersonic airfoils, *J. aero. Sci.* **23,** 4, 398–399 (Readers' Forum), Apr. 1956.

Formulas are given for the calculation of aerodynamic forces and moments on flat-plate and double-wedge airfoils in supersonic and hypersonic flows. It is stated that the formulas are based on curve-fitting numerical data obtained from linear theory, shock-expansion hypersonic theory, and the method of characteristics; but very few additional details are supplied.

J. R. Spreiter, USA

2983. Bobbitt, P. J., Linearized lifting-surface and lifting-line evaluations of sidewash behind rolling triangular wings at supersonic speeds, *NACA TN* 3609, 63 pp., Mar. 1956.

Author considers problem of determining sidewash behind a rolling triangular wing in supersonic flow, where wing leading edge is subsonic. Analysis is on the basis of linearized supersonic wing theory. Sidewash is first calculated by differentiation of the velocity potential which is the solution of the boundary-value system the problem presents. Both analytical and numerical calculations are formidable in this approach. Following procedures suggested for downwash calculations [J.C. Martin, AMR **4,** Rev. 1649; H. Mirels and R.C. Haefeli, AMR **4,** Rev. 2645; **3,** Rev. 2404], author replaces lifting surface by a finite set of skewed horse-shoe vortexes. Comparison of the results of the two methods are made for various Mach numbers, wing semi-apex angle, and downstream and vertical stations (in wing plane of symmetry). Good agreement is shown save in the vicinity of trailing edge when leading edge is nearly sonic.

W. H. Pell, USA

2984. Licher, R. M., Reduction of supersonic wave drag for wing-fuselage combinations at zero lift, *Douglas Aircr. Co. Rep.* SM-19436, 13 pp., Nov. 1955.

Paper considers optimum fuselage shaping for fixed wing geometry, applying previously determined criteria to wings of elliptic planform. As stated, the general problem is not well defined and only in subsequent analysis are constraints on total volume and nature of modifications clarified. In the applications, no discussion is given of the importance of blanketed (within fuselage) portion of wing or of final shape of configuration.

Orientation of derived results to other research on reduction of

supersonic wave drag would seem to have been appropriate; its absence is presumably attributable to classification imposed until recently on the original contributions in this field.

M. A. Heaslet, USA

2985. **Etkin, B., Lift distribution on warped supersonic wings,** *Canad. aero. J.* **1,** 1, 16–20, Apr. 1955.

Paper gives a process of numerical integration, in a form especially convenient for automatic computers, to calculate the perturbation potential (also its derivative in direction of undisturbed flow) on the wing surface. Paper is based on linearized supersonic wing theory. It is assumed that the thin wing has supersonic trailing edges and at least one supersonic leading edge, so that the upwash field in front of the wing and in its plane is easily taken care of since it is either known or can be eliminated with the aid of Evvard's equivalent-area theorem. The method is essentially that given by V. Linnaluoto in *SAAB TN 6* (1952) and unnecessarily confined by that author to the case of nonlifting symmetrical wings with subsonic leading edges.

H. Behrbohm, Sweden

2986. **Gallagher, J. J., and Mueller, J. N., An investigation of the maximum lift of wings at supersonic speeds,** *NACA Rep.* 1227, 25 pp., 1955.

Various wing planforms of random thickness distribution were tested at Mach numbers of 1.55, 1.90, and 2.32. In general, the maximum lift coefficient was approximately 1.05 ± .05. It appeared to be independent of planform and decreased slightly with increasing Mach number. The lift curves remained linear for angles of attack as high as 20° to 30° and lift-drag ratios at maximum lift were of the order of 1.0.

From authors' summary by A. Petroff, USA

2987. **Gandel´sman, A. F., Gukhman, A. A., and Ilyukhin, N. V., Investigation of the drag coefficient variation for supersonic gas flows** (in Russian), *Teploenergetika* **2,** 1, 17–23, 1955 (translated from Russian by M. D. Friedman, 572 California St., Newtonville, Mass., 18 pp.).

Paper studies variation in the relationship of the reversible and irreversible parts of the total pressure drop along the supersonic part of a nozzle. The one-dimensional flow model is used for the basis of the investigation. Supersonic flow being obtained in a diverging nozzle of small conicity, this permits deviation of local stream direction to be neglected; and for any specific pressure drop, the length of channel is increased as the cone angle is decreased and test length is less than 8 calibers.

A computational method is explained, and the results of tests are compared with calculated values, good agreement being obtained.

A. F. W. Langford, Australia

2988. **Burbank, P. B., A method for calculating the contour of bodies of revolution with a prescribed pressure gradient at supersonic speed with experimental verification,** *NACA TN* 3555, 64 pp., Mar. 1956.

Method of characteristics is used to calculate the contour of a sharp-nosed body of revolution having a prescribed axial pressure distribution. After checking shape of a known parabolic body, author applies his method to design a body whose axial pressure distribution is a linear function of axial distance from the nose with initial cone angle of 30° for a free-stream Mach number of 3.13. Measured pressure distributions on a model at $M = 3.05$ and $M = 3.13$ agreed well with theory. Such bodies are not expected to show constant pressure gradients at off-design Mach numbers.

R. E. Street, USA

2989. **Jones, C. W., Elements of an improved linear perturbation theory of steady supersonic flow with axial symmetry,** *Proc. Camb. phil. Soc.* **52,** 2, 336–343, Mar. 1956.

In the frame of the usual linearized theory, the terms uv and v^2/a^2 of the equation for the potential are neglected and the term u/a is replaced by the Mach number M of the undisturbed stream. In this paper, author generalizes the usual linearized theory by approximating the terms u^2/a^2, v^2/a^2 and $u\,v$ by mean values. In this way, one obtains a linear equation for the potential that is a little more general. The Mach lines are straight and parallel again but differ from the Mach lines of the undisturbed flow. The general solution of the equation for the potential is given by a series, the terms of which are products of powers and hypergeometric functions.

A numerical example (10° cone in a stream of Mach number $M = 3.323$) shows that the new formulas give a better approximation of the velocity field in the neighborhood of the body than the usual linearized theory; furthermore, it is possible to fit a shock of constant strength to the improved solution.

Finally, author makes the tentative suggestion that a technique due to G. B. Whitham perhaps will lead to an improved description of the whole field of flow.

R. Sauer, Germany

2990. **Lomax, H., Wing-body combinations with certain geometric restraints having low zero-lift wave drag at low supersonic Mach numbers,** *NACA TN* 3667, 32 pp., Feb. 1956.

When untwisted and uncambered wings are mounted centrally on cylindrical tubes, the optimum fuselage area distribution giving minimum wave drag is determined, when the fuselage ends at some plane and when the fuselage diameter is fixed at various points. The interference between the central portion of the "airplane" and its nose and tail region being neglected in the above-mentioned problems, the entire fuselage is also considered, assuming that it is a slender closed body of revolution.

The indentation of the body is determined such as to yield minimum wave drag, when different kinds of geometrical restraints are imposed on the body or the combination of wings and body. The geometrical restraints are: (1) Unlimited indentation length, fixed total volume, the body being long enough to contain the apexes of the Mach forecone and aftercone enclosing the wings; (2) limited indentation length on Sears-Haack body, fixed total volume, the body being long enough to contain the apexes of the Mach forecone and aftercone enclosing the wings; (3) limited indentation length on arbitrary body, fixed total volume; (4) limited indentation on arbitrary body, the cross-sectional area of the body being fixed at some intermediate station.

E. M. de Jager, Holland

2991. **Friedman, M. D., and Cohen, Doris, Arrangement of fusiform bodies to reduce the wave drag at supersonic speeds,** *NACA TN* 3345, 23 pp., Nov. 1954.

Using linearized slender-body theory and reverse-flow theorems, the wave drag of a system of fusiform bodies at zero angle of attack and supersonic speeds is studied to determine the effect of varying the relative location of the component parts. The investigation is limited to two-body and three-body arrangements of Sears-Haack minimum-drag bodies. It is found that in certain arrangements the interference effects are beneficial, and may even result in the two- or three-body system having no more wave drag than that of the principal body alone. The most favorable location appears to be one in which the maximum cross section of the auxiliary body is slightly forward of the Mach cone from the tail of the main body. The least favorable is the region between the Mach cone from the nose and the forecone from the tail of the main body.

From authors' summary

2992. **Wedderspoon, J. R., and Young, A. D., Note on the results of some profile drag calculations for a particular body of revolution at supersonic speeds,** *Coll. Aero. Cranfield Rep.* no. 81, 20 pp. + 15 figs., July 1954.

Details additional to those discussed in *Coll. Aero. Cranfield Rep.* no. 73 are given of the method developed for the calculation of the profile drag of bodies of revolution at supersonic speeds

and zero incidence. The method has been applied to a particular body of fineness ratio 7.5 for Mach numbers ranging from 1.5 to 5.0, Reynolds numbers ranging from 10^6 to 10^8, and transition positions ranging from the nose to the tail end of the body. The calculations assume zero heat transfer. Results indicate that the over-all difference in profile drag between fully laminar and fully turbulent flow decreases rapidly with mainstream Mach number and rather more rapidly than does the corresponding difference for a flat plate; at Mach numbers greater than about 2, the profile drag of the body with fully turbulent flow is less than that of a flat plate. From authors' summary

2993. Deich, M. E., Flow of gas through turbine lattices, *NACA TM* 1393, 136 pp., May 1956. (Translation of: Tekhnickeskaga gazodinamika, Chap. 7, 1953)

Paper is a translation of chap. 7 of the book "Technical gas-dynamics" [see AMR **9**, Rev 1869]. The topics treated are best shown by the list of paragraph headings. They are: 7-1. Geometrical and gasdynamical parameters of the lattices; fundamentals of flow through lattices; 7-2. Theoretical methods of investigation of plane potential flow of incompressible fluid through a lattice; 7-3. Electro-hydrodynamic analogy; 7-4. Forces acting on an airfoil in a lattice; Theorem of Joukowsky for lattices; 7-5. Fundamental characteristics of lattices; 7-6. Friction losses in plane lattice at subsonic velocities; 7-7. Edge losses in plane lattice at subsonic velocities; 7-8. Several results of experimental investigations of plane lattices at small subsonic velocities; 7-9. Flow of gas through lattice at large subsonic velocities; Critical Mach number for lattice; 7-10. Profile losses in lattices at large subsonic velocities; 7-11. Flow of a gas through reaction lattices at supersonic pressure drops; 7-12. Impulse lattices in supersonic flow; 7-13. Losses in lattices at near sonic and supersonic velocities; 7-14. Computation of angle of deflection of flow in overhang section of a reaction lattice at supersonic pressure drops; 7-15. Characteristic features of three-dimensional flow in lattices. N. Tetervin, USA

Wave Motion in Fluids

(See Revs. 2955, 3108)

Turbulence, Boundary Layer, etc.

(See also Revs. 2955, 2975, 3023, 3046, 3063, 3074, 3077, 3093, 3125, 3137)

2994. Yuan, S. W., and Finkelstein, A. B., Laminar pipe flow with injection and suction through a porous wall, *Trans. ASME* **78**, 4, 719–724, May 1956.

Incompressible fluid flows through a circular porous pipe, the fluid velocity normal to the wall being constant. A similarity transformation reduces the Navier-Stokes equations to a third-order ordinary differential equation which is solved by a perturbation method for both large and small flows through the porous wall. Detailed numerical results show that both the pressure drop and skin friction are increased by small amounts of fluid injection. Converse applies for suction. The results are relevant to such problems as the cooling of combustion chambers by fluid injection, and the ducting associated with porous suction used for boundary-layer control. D. G. Hurley, Australia

2995. Sinha, K. D. P., The laminar boundary-layer with distributed suction on an infinite yawed cylinder, *Aero. Res. Counc. Lond. curr. Pap.* 214, 36 pp. + 19 figs., 1956.

Author has calculated spanwise velocity profiles within the boundary layer of infinite yawed cylinder for those cases where solutions for the chordwise flow (which can be calculated independently) are available. The chordwise external flow is of the Falkner-Skan type ($U = cx^m$), which leads to an ordinary differential equation and admits of a boundary condition corresponding to a definite suction distribution depending only on a proportionality factor. The spanwise flow depends on the chordwise solution but requires only solution of a linear differential equation. For the case $m < 0$, author has calculated the spanwise flow for 5 suction factors and 4 values of m, besides supplying exact solutions for $m = -1/3$ and $m = -1$. For each value of the suction factor, the velocity profiles are found similar when plotted against the normal coordinate divided by spanwise displacement thickness. Further, the ratio of spanwise displacement thickness to spanwise momentum thickness is found almost independent of m, but varying with the suction factor. For $m > 0$, only few chordwise solutions with suction being available, the given spanwise solutions are equally scarce. The conclusions, however, are similar. A. von Baranoff, France

2996. Lachmann, G. V., Boundary layer control, *J. roy. aero. Soc.* **59**, 531, 163–198, Mar. 1955.

2997. Kuethe, A. M., Some features of boundary layers and transition to turbulent flow, *J. aero. Sci.* **23**, 5, 444–452, 506, May 1956.

This is a useful survey of some recent work on transition and turbulent flows in the pressure of a solid boundary. Especially welcome is a preliminary account of some new unpublished measurements on pipe transition induced by an oscillating ring airfoil in a Hagen-Poiseuille flow (U. of Michigan dissertation, supervised by the author). S. Corrsin, USA

2998. Bertram, M. H., Exploratory investigation of boundary-layer transition on a hollow cylinder at a Mach number of 6.9, *NACA TN* 3546, 38 pp., May 1956.

The Reynolds number for transition on the outside of a hollow cylinder with heat transfer from the boundary layer to the wall has been investigated at a Mach number of 6.9 in the Langley 11-in. hypersonic tunnel. From consideration of only the results obtained in that portion of the nozzle which surveys indicated to have an essentially constant Mach number, the Reynolds number for the start of transition was between 4×10^6 and 6×10^6. From a correlation of results obtained from various sources at lower Mach numbers (in the range 2.0 to 4.5), leading-edge thickness and free-stream Reynolds number per inch appear to be important considerations in flat-plate transition results. At a given Mach number it appears that the Reynolds number based on leading-edge thickness is an important parameter that must be considered in comparisons of flat-plate transition data from various facilities. From author's summary

2999. Sinclair, A. R., and Czarnecki, K. R., Investigation of boundary-layer transition on 10° cone in Langley 4- by 4-foot supersonic pressure tunnel at Mach numbers of 1.41, 1.61, and 2.01, *NACA TN* 3648, 17 pp., May 1956.

An investigation has been made to determine the transition Reynolds numbers on a 10° cone in the Langley 4- by 4-foot supersonic pressure tunnel at Mach numbers of 1.41, 1.61, and 2.01 and over a Reynolds number range from about 0.8×10^6 to 9.5×10^6 per foot. The results indicated that the transition Reynolds numbers increased with tunnel stagnation pressure and that there was no effect of Mach number on transition Reynolds numbers. A single-element surface roughness caused a larger decrease in transition Reynolds number than was experienced in low-speed or in other supersonic wind-tunnel investigations. From authors' summary

3000. Steinman, H., The stability of viscous flow between rotating cylinders, *Quart. appl. Math.* **14,** 1, 27–33, Apr. 1956.

Chandrasekhar's method [S. Chandrasekhar, *Mathematika* **1,** 5] of solving the characteristic value problem is extended to the second order in y/d, where y is the distance of a point from the inner cylinder and d is the difference in radii of the two cylinders. Calculated critical angular velocities are in closer agreement with Taylor's experimental results [G. I. Taylor, *Phil. Trans. roy. Soc. Lond.* (*A*) **223,** 289] and are approximately 4% larger than those velocities calculated using only first-order terms. Author's second-order calculations and Taylor's calculations give results agreeing about equally well with Taylor's experimental results for nine compared cases. Present method is more simple and more general, however, than Taylor's theoretical method.

E. L. Knuth, USA

3001. Einstein, H. A., and Li, H., The viscous sublayer along a smooth boundary, *Proc. Amer. Soc. civ. Engrs.* **82,** EM 2 (*J. Engng. Mech. Div.*), Pap. 945, 27 pp., Apr. 1956.

A new type of flow is visualized for the viscous sublayer. Authors describe a periodic growth and collapse of the layer and review experimental evidence which tends to prove that this is a more exact description than steady flow. Mathematical equations are derived for the velocity distribution through the sublayer and results are compared to measurements. Intensity of turbulence and viscous dissipation are also computed and found to compare favorably with measurements. Authors present data from instantaneous pressure measurements and show that these involve a periodic component comparable to their theoretical period. An appendix gives a description of the apparatus and procedure used for the measurements of the pressure fluctuations.

W. D. Baines, Canada

3002. Krahn, E., The laminar boundary layer on a rotating cylinder in crossflow, *U.S. nav. Ord. Lab. Rep.* 4022, 19 pp., June 1955.

Paper is concerned with the dependence of the circulation in the potential flow around a cylinder on the rotational speed of the cylinder. The answer to this problem is sought by calculating the laminar boundary layer. Two approximate methods are used. One is due to Burgess and the other is an adaptation of the Pohlhausen method. Velocity profiles and shearing stress are computed only in the case of one stagnation line on the surface of the cylinder.

S. Ostrach, USA

3003. Yen, K. T., On the energy balance in a compressible boundary layer, *J. aero. Sci.* **23,** 3, 274–276(Readers' Forum), Mar. 1956.

Author considers energy balance in a two-dimensional, steady, compressible, turbulent boundary layer consisting of a mean flow and its turbulent fluctuations. For the case of flow over an insulated flat plate and Prandtl number equal to unity, it is shown that the total temperature of the mean flow differs from a constant by the fluctuations of the velocity components.

A. Ritter, USA

3004. Stuart, D. J. K., Analysis of Reynolds number effects in fluid flow through two-dimensional cascades, *Aero. Res. Counc. Lond. Rep. Mem.* 2920, 28 pp. + 20 figs., 1955.

Experimental investigation into the effects of Reynolds number on two-dimensional cascade flow was conducted and results analyzed. It is pointed out that the main loss of efficiency of an axial compressor is due to boundary-layer separation over the cascade foil. At low Reynolds number, separation occurs in laminar regime; at higher Reynolds number, it occurs in turbulent regime. Although transition from laminar to turbulent boundary layer is inevitably accompanied by a higher skin-friction coefficient, the loss resulting from turbulent separation is usually lower than from laminar separation due to smaller wake size involved. Thus there exists an optimum range of Reynolds number within which each axial compressor should operate. For an efficient design, good estimation of transition point is vital. To insure best performance, transition must be made as near to the laminar separation as possible. Too late transition usually results in unduly large loss due to completed laminar separation, while too early transition usually results in losses due both to increased skin friction in turbulent boundary layer and to the advance of turbulent separation. As both transition and separation depend on the external potential flow, best pressure distribution for laminar and turbulent boundary layers are discussed

Good agreement between theoretical prediction and experimental results are noted.

H. S. Tan, USA

3005. Acharya, Y. V. G., Momentum transfer theory applied to a turbulent jet spreading in still air, *Appl. sci. Res.* (*A*) **5,** 4, 265–267, 1955.

Aerodynamics of Flight; Wind Forces

(See also Revs. 2827, 2834, 2905, 2913, 2942, 2963, 2968, 2979, 2981, 2982, 2983, 2984, 2985, 2986, 2987, 2990, 2993, 3024, 3026, 3028, 3038, 3045)

3006. Sears, W. R., Some recent developments in airfoil theory, *J. aero. Sci.* **23,** 5, 490–499, May 1956.

Author reviews extensions of classical airfoil theory to problems involving viscous effects. Review is concerned with (1) the theory of profiles with boundary layers in steady and unsteady flow, including extensions of unsteady airfoil theory; (2) the theory of wings with leading-edge separation; (3) Prandtl wing theory applied to partially stalled wings. Attention is drawn to the need for further investigation of the above problems which are of considerable practical importance.

W. P. Jones, England

3007. Powell, J. B. L., The effect of dihedral on the lift and drag coefficients of airfoils in supersonic flow, *Quart. J. Mech. appl. Math.* **9,** 1, 51–74, Mar. 1956.

Pressure distributions and lift and drag coefficients are calculated for wings with dihedral by using linearized theory. The wings considered are those for which conical-field theory is applicable, and the results can be superposed to give the flow past more general shapes. Detailed results for variations of dihedral and leading edge sweep are given as graphs. For delta wings with dihedral, lifts for dihedral α are compared with the result of multiplying the corresponding lift for $\alpha = 0$ by $\cos^2\alpha$, and it is found that the percentage difference is less than 0.7% for $\alpha = 10^\circ$ and 8% for $\alpha = 30^\circ$ (the greatest difference occurs when the leading edge is swept at angles near to the Mach angle), which should be a very valuable rule in applications.

G. N. Ward, England

3008. Love, E. S., Investigations at supersonic speeds of 22 triangular wings representing two airfoil sections for each of 11 apex angles, *NACA Rep.* 1238, 60 pp., 1955.

Report contains the results of experimental measurements of the lift, drag, pitching moment, and pressure distributions on two series of eleven triangular wings at Mach numbers of 1.62, 1.92, and 2.40. The two series of wings had identical planforms, a constant thickness ratio of 8%, a constant maximum thickness point at 18% chord, and a range of apex half-angles from 10° to 45°. The first series had an elliptical leading edge and the second series, a wedge leading edge.

It was concluded that the aerodynamic gains experienced by the elliptical-leading-edge wings as compared to the wedge-leading-edge wings was due principally to the favorable effect of the gentle curvature of the ridge line common to the elliptical-leading-edge shape.

J. Persh, USA

3009. Brebner, G. G., The calculation of the loading and pressure distribution on cranked wings, *Aero. Res. Counc. Lond. Rep. Mem.* 2947, 25 pp. + 1 table + 14 figs., 1955.

To avoid unsatisfactory stalling and pitching-moment characteristics and structural difficulties due to elastic distortions of swept wings, so-called "cranked wings" (i.e., wings with discontinous changes of sweep; *M* and *W* planforms) are developed. Using distributions of vortexes and sources over the airfoil surface, author extends former work [Küchemann: *R. & M.* 2935] to approximate formulas for finding the spanwise and chordwise pressure distribution at the crank in incompressible flow. Experimental results are in good agreement herewith. The new method seems superior to an earlier method with regard to quality and speed of calculation. H. Thieme, Germany

3010. Schneider, W. C., A comparison of the spanwise loading calculated by various methods with experimental loadings obtained on a 45° sweptback wing of aspect ratio 8.02 at a Reynolds number of 4.0 × 10⁶, *NACA Rep.* 1208, 11 pp., 1954.

Experimental force and moment data obtained by pressure measurements on a wing of aspect ratio 8.02, 45° sweepback of the quarter-chord line, taper ratio of 0.45, and NACA 63_1A012 airfoil sections parallel to the plane of symmetry have been compared with the calculated loadings obtained by the standard methods proposed by Weissinger, Falkner, and Multhopp, as well as by variations thereof. On the basis of the present calculations, the Weissinger solution, when the number of control points was increased from 7 (the number suggested by Weissinger) to 15, or the Multhopp solution when using at least 15 control points, resulted in a good compromise between lift-curve slope and load shape, although all of the calculations underestimated the lift-curve slope. From author's summary

3011. Hamal, Y., Modification of the aerodynamic properties of a wing in the neighborhood of the ground (in French), *Belgian roy. Soc. Engrs. Indust., Centre nat. Etudes Rech. aero. Mem.* **4,** 54 pp., 1953.

Paper reviews theoretical methods for determining effect of ground on aerodynamic characteristics of wings in incompressible flow. Numerical examples are worked out. Wind-tunnel techniques (including wall corrections) for ground effect are discussed. Author's experimental results are compared with theory. Disagreement is attributed to method of representing ground in wind tunnel (i.e., use of groundboard). W. L. Haberman, USA

3012. Miles, J. W., The aerodynamic force on an airfoil in a moving gust. A generalization of the two-dimensional gust problem, *Douglas Aircr. Co. Rep.* SM-18598, 32 pp., Oct. 1954.

This investigation was motivated by the problem presented by a winged helicopter in which the blade shaft was located near the wing tip. The velocity field induced by the wing presents a gust profile which moves across the helicopter blade section at a velocity different from the blade's flight velocity. The incremental loading on the blade section resulting from passage through this gust is calculated assuming incompressible flow. The separation of response into "circulation" and "virtual mass" components is shown by developing the solution from Schwarz' basic result for the pressure on an oscillating airfoil having an arbitrary chordwise distribution of upwash. R. M. Spath, USA

3013. Foss, K. A., Effects of structural flexibility on gust loading of aircraft. Part I. Gust loads on swept-wing airplanes free to pitch and to deform statically, *WADC TR* 54-592, part I, xi + 104 pp., Mar. 1955.

Report presents elaborate calculation of typical gust load case for rigid or "equivalent rigid" example swept-wing airplane (sweep 35°, aspect ratio 9.4), free to move with the gust and to pitch. Problem inevitably requires long series of approximations and assumptions to obtain workable representations of aerodynamic forces involved. Procedures adopted are, as a rule, conventional for this type of work and mostly evolve from strip theory operating with Wagner-Küssner unsteady aerodynamics. Unsteady wing downwash at tail is derived by separate calculation, and slender body result is used for forces on fuselage. Equations of motion are solved by step-by-step integration for sharp-edged gust case. Generalizations to other gust forms are derived by superposition principle.

Issue must be said to remain partly questionable, more so than in the aerodynamically far simpler straight-wing case. Several checks are not convincing because they do not refer to stresses, but to c.g. acceleration only, which should not always be the effective criterion. Conclusion reached is that pitching degree of freedom is relatively important in swept-wing case. The most aft c.g. position gives rise to the greatest c.g. acceleration.

Report is important to those planning similar calculations; it does not present systematic results suitable for general use. J. H. Greidanus, Holland

3014. Kirsch, A. A., Calligeros, J. M., and Foss, K. A., Effects of structural flexibility on gust loading of aircraft. Part II. Dynamic stresses in swept-wing airplanes, *WADC TR* 54-592, part 2, xi + 189 pp., Aug. 1955.

Gust-load calculation established in part I [see preceding review] is extended by introducing five additional degrees of freedom: fuselage bending, first wing bending, first wing torsion, first horizontal tail bending, and first horizontal tail torsion, each represented by proper generalized coordinate and assumed specific cantilever deformation. Equations of motion are obtained by Lagrange method. The aerodynamics is similar in principle to that of part I. Solution is obtained by numerical step-by-step procedure.

Example swept-wing airplane is the same as that used in part I. Computations are, in effect, performed only for four degree-of-freedom selections from the unworkable abundance initially introduced.

Conclusions are that fundamental wing bending is essential for proper evaluation of wing stresses, that the alleviation commonly attributed to flexible sweptback wings is partly cancelled by increased pitching motion, and that dynamic overstress estimates from straight-wing analyses are not applicable to sweep case.

Appendix on convergence of transient stress methods is largely unclear to reviewer. J. H. Greidanus, Holland

3015. Coleman, W. S., Stabilization of the aeroplane in symmetric flight at zero or very small air speeds, *Aero. Quart.* **6,** 4, 295-328, Nov. 1955.

Paper is concerned with longitudinal stability of VTOL and STOL aircraft in take-off and landing. Assumptions are that damping stability derivatives are negligible and automatic reaction-type stabilization, giving moments proportional to pitch and pitch-rate, will be used. Solutions for oscillatory vertical gust are obtained in closed form, assuming stability adequate to consider C_m constant. Simplified step-by-step method is used to solve more general cases. A number of example calculations are presented. Results for simplified hovering cases are shown to have considerable significance for more general unsteady cases. A. A. Schy, USA

3016. Curfman, H. J., Jr., Theoretical and analog studies of the effects of nonlinear stability derivatives on the longitudinal motions of an aircraft in response to step control deflections and to the influence of proportional automatic control, *NACA Rep.* 1241, 21 pp., 1955.

Author finds that the effect of two nonlinear stability derivatives, $C_m(\alpha)$ and $C_L(\alpha)$, on the longitudinal motions of an aircraft may be compensated for by adjusting the autopilot proportionality constant to assure stability. For the nonlinear pitching moment variations, step deflections in elevator surface cause aperiodic oscillations. For the nonlinear lift curve, only damping of the transient angle of attack oscillations was affected.

This paper, dated 1950 and published in 1955, is now slightly outmoded in technique since a great deal of nonlinear equipment has been added to analog flight simulators. However, the basic treatment of the effects about sharp segment break points is worthy of mention. Both of the derivatives should have been defined so that $dC_m/d\alpha$ and $dC_L/d\alpha$ exist at these breakpoints ($\pm 2°$).
A. C. Block, USA

3017. Shinbrot, M., On the analysis of linear and nonlinear dynamical systems from transient-response data, *NACA TN* 3288, 41 pp., Dec. 1954.

It is noted that various techniques for calculation of coefficients of differential equations of motion of aircraft from measured transient response data have in common a set of numerical integrations of the equations, after they have been multiplied by N weighting or "method" functions, followed by a least-squares procedure. It is then pointed out that these weighting functions need not be connected with specific device such as the LaPlace transform but should, in fact, be optimized in some obvious ways to bring out full accuracy of all the data. A particular optimization is described in detail, and some examples are given which bring out fact that procedure may be applied to nonlinear systems.
L. Becker, USA

3018. Kerr, T. H., A criterion for the prediction of the recovery characteristics of spinning aircraft, *Aero. Res. Counc. Lond. curr. Pap.* no. 195, 14 pp. + 8 figs., 1955.

In an attempt to establish a simple criterion for the prediction of the spin and recovery characteristics of aircraft, it has been deduced that the two most important parameters are the unbalanced rolling moment coefficient about the wind axis in the spin and the ratio of pitching to rolling moment of inertia. Using the results of full-scale spinning tests on thirty-three aircraft, it has been possible to establish empirical relationships between the estimated unbalanced rolling moment coefficient and the inertia ratio which effectively divide the aircraft into the three groups which have satisfactory, borderline, and unsatisfactory recovery characteristics.

A simple method is presented for estimating the unbalanced rolling moment coefficient if only the shape of the aircraft is known. With this information and a knowledge of the mass distribution of the aircraft, the empirical relationships should give a good indication of the spin recovery characteristics on new designs. This method is expected to be of particular value to aircraft designers in the early design stages, since the method does not depend on the result of tunnel tests. From author's summary

3019. Santangelo, G., On the vertical take-off and vertical climb of aircraft (in Italian), *Aerotecnica* **35,** 1, 3–10, Feb. 1955.

3020. Hamilton, J. A., Full scale measurements of impact loads on a large flying boat. Part II-Results for impacts on main step, *Aero. Res. Counc. Lond. curr. Pap.* no. 205, 29 pp. + 24 figs., 1955.

Full-scale measurements of landing impact forces and pressures have been made on the hull of a Sunderland Mk.5 flying boat (weight 50,000 lb) in order to provide basic information on the agreement between experiment and the latest available impact theories. The experiments were arranged to give impact conditions as near as possible to those assumed in theory.

The results revealed a marked discrepancy between the form of the total impact-time curve predicted by theory and that measured on the aircraft. In particular, the measurement of time to reach maximum impact force was about twice that indicated by theory. The magnitudes of the theoretical and measured maximum impact forces were in reasonable agreement. This discrepancy may be attributable to the neglect of after body influences in the theoretical analyses. From author's summary

3021. Sinnott, C. S., Hinge-moment derivatives for an oscillating control, *Aero. Res. Counc. Lond. Rep. Mem.* 2923, 11 pp., 1955.

The semi-empirical "equivalent profile" method of W. P. Jones (1948) is extended to the case of an airfoil with an oscillating control. The oscillatory hinge-moment derivatives for such an airfoil-control combination in a low-speed wind tunnel are estimated, an allowance for tunnel wall interference effects being included.

A comparison with measured values of the control derivatives is made for two values of the control-chord ratio, representing an aileron and a tab. The method of this report gives results in much better agreement with experiment than those obtained by vortex-sheet theory. From author's summary

3022. Wallis, R. A., and Ward, G. F., Determination of optimum air jet arrangement to delay nose stalling on the articulated nose of a thin wing, *Aero. Res. Labs. Melbourne, Austral.* AN 133, 8 pp. + 5 figs., Jan. 1954.

The optimum air jet configuration for suppressing laminar separation on the nose of a 10-ft chord thin wing was determined. A single row of small holes midway between the stagnation point just prior to nose stalling and the leading edge at a pitch of ¼ in. gave best results. From authors' summary

3023. Van Every, K. E., Selected aerodynamic problems of high-speed flight, *AGARD Publications* AG 14/P 5, 99–107, May 1954.

In a recent survey of aerodynamic developments in Western Europe, a number of common difficulties appeared in flight testing high-speed aircraft. In general, these difficulties can be ascribed to one or more of three basic types of aerodynamic phenomena: (1) Boundary-layer separation, particularly in combination with local shock waves; (2) nonstationary flow; (3) transonic flow.

The purpose of this paper is to give a general idea of the status of the work being done on these problems and to suggest what else needs to be done. From author's summary

Aeroelasticity (Flutter, Divergence, etc.)

(See also Revs. 3013, 3014, 3021, 3036)

3024. Wang, C.-T., Vaccaro, R. J., and De Santo, D. F., A practical approach to the probelm of stall flutter, *Trans. ASME* **78,** 3, 565–572, Apr. 1956.

Authors start out with a refreshingly simple and sound basic approach, using the single-degree-of-freedom torsional flutter equation, lumping the aerodynamic movement into a single complex quantity $C_m = a_m + ib_m$. Neglecting structural damping, the condition of flutter is $b_m = 0$. The real part a_m and critical flutter frequency k are determined from tests on a fluttering single airfoil. The a_m quantity appears to have a minor influence on k, that is, flutter frequency is rather close to natural torsional frequency, a fact not too surprising in the range of blade-mass-to-air-mass ratio valves encountered for compressor blades. Authors proceed to demonstrate that the results from single-degree-of-freedom tests may be used to predict two-degrees-of-freedom results with respect to torsional flutter.

In this reviewer's opinion, authors' approach appears very promising. Since most severe cases of blade failures actually appear to be the result of bending vibrations in the stall region, reviewer would suggest an extension of authors' method also to this case, though it might be that reviewer is overlooking difficulties in accounting for neighbor blades. J. R. Schnittger, Sweden

3025. Nickel, K., Airfoil system in two-dimensional flow with arbitrary non stationary movements (in German), *Ing.-Arch.* **23,** 3, 179–188, 1955.

Author extends the Birnbaum thin-airfoil theory of a single airfoil moving unsteadily to systems of airfoils with parallel chordlines. Systems of integral equations for vortex distributions over airfoils (Fredholm type) and for trailing vorticity behind the airfoils (Sonine type) are established. General solutions are not given. Only for two special cases of airfoil systems, i.e., a cascade with angle of stagger of 90° and a row of airfoils lying behind each other, respectively, are solutions of Fredholm equations given. However, formulas for vortex distributions, lift, and pitching moment still contain the unknown solutions of Sonine integral equations. The theorem, that for a single airfoil moving through an unstationary gust the chordwise lift distribution is proportional to the lift distribution of a flat plate in stationary flow, can be extended to systems of airfoils. K. Gersten, Germany

3026. Pinsker, W. J. G., A note on the dynamic stability of aircraft at high-subsonic speeds when considering unsteady flow, *Aero. Res. Counc. Lond. Rep. Mem.* 2904, 28 pp., 1954.

The effect of an increase in speed relative to the speed of sound on the unsteady flow round a harmonically oscillating airfoil, is to increase the lag of the aerodynamic forces and moments behind the deflection when the frequency is small. It is shown theoretically that this will result in a serious deterioration of the damping of both the lateral oscillation and the high frequency longitudinal oscillation with high Mach numbers. Use is made of derivatives calculated for flutter purposes to estimate the unsteady derivatives at aircraft oscillation frequencies. Illustrative examples are presented. From author's summary

3027. Schultze, E., The response of airplane wings to externally applied forces (in German), *ZAMP* **6,** 2, 126–135, 1955.

Using the theory of linear integral equations of the second kind, paper investigates the steady-state response of wings due tc sinusoidally varying shaker forces. The effect of complex damping (i.e., damping proportional to amplitude and in phase with velocity) is included. Particular attention is paid to the problem of exciting pure natural modes. It is shown that for a fixed position of the shakers and an appropriate ratio of the force amplitudes it is possible to eliminate at least $(n-7)$ natural modes. Still more modes can be suppressed by varying the position of the shakers. Finally, the author presents a mathematical proof for the experimental procedure of Lewis and Wrisley [AMR **4,** Rev. 1460]; this procedure, contrary to the author's method, requires a large number of shakers to excite pure natural modes.

J. H. Argyris, England

3028. Chang, C.-C., and Chu, W.-H., Aerodynamic interference of cascade blades in synchronized oscillation, *J. appl. Mech.* **22,** 4, 503–508, Dec. 1955.

Present paper is essentially a welcome extension of the authors' earlier paper on this subject [AMR **9,** Rev. 1193]. It is strictly a theoretical attack on the problem. The reviewer's opinion on authors' method is unchanged.

J. R. Schnittger, Sweden

3029. Wang, C.-T., Lane, F., and Vaccaro, R. J., An investigation of the flutter characteristics of compressor and turbine blade systems, *J. aero. Sci.* **23,** 4, 335–344, Apr. 1956.

Authors deal with flutter in the unstalled region. Their work appears to be an interesting and careful theoretical and experimental investigation, constituting a broadening and an extention of ideas also recently advanced by Sisto. The complexity of a row of vibrating compressor blades is reduced to an equivalent single blade by introducing the assumption that all blades must oscillate with identical blade mode, at identical amplitude, and with some undetermined inter-blade phase angle σ, characterizing the system mode shape. An interesting mathematical justification for this assumption is presented by means of matrix calculus. The parameter σ is perhaps the most important single quantity studied. Theoretical results for staggered and unstaggered cascades are presented and compared to classical, single-airfoil values of critical frequency k and frequency ratio ω_α/ω. Experimental results in good agreement with theoretical results are given for zero stagger cascade at zero incidence and include k, amplitude ratio, and inter-blade phase angle versus mass unbalance.

J. R. Schnittger, Sweden

Propellers, Fans, Turbines, Pumps, etc.

(See also Revs. 2844, 2961, 2993, 3022, 3025, 3029, 3070, 3103, 3116, 3144, 3145)

Book—3030. Smith, C. W., Aircraft gas turbines, New York, John Wiley & Sons, Inc., 1956, xv + 448 pp. $8.75.

Present volume is part of a series written by members of the General Electric Company's staff on a wide variety of subjects with the objective of advancing engineering practice. The character of the book is partly derived from the editorial policy of the series, which apparently requires a presentation which is understandable to readers of elementary education. The aircraft gas turbine is a device which was made possible by drawing on practically all of the resources of our machine civilization. It represents the supreme achievement among our prime movers, and many of its components, such as the compressor and the combustion chamber, still present obscure and unsolved problems. To present the aspects of design and testing of this device on the elementary level selected for this series is indeed a difficult undertaking; on the whole, the author has succeeded well. Criticisms of certain aspects of the book are perhaps more appropriately to be regarded as a criticism of the basic plan rather than of its achievement.

In eighteen chapters and a little over four hundred pages, the essential aspects of the aircraft gas turbine are covered from the basic elements of fluid mechanics, thermodynamics, through consideration of the design problems of the components, stresses, and materials, to performance and installation.

Reviewer misses a sense of the historical perspective which is so important in understanding the real achievement of this development. He misses some illustrative examples of mechanical constructions. In return, he would be willing to forego certain tedious and not always wholly illuminating reviews of basic principles of mechanics. The only chapter which seems inadequate is the one entitled, "Stresses and materials," which does not appear to do justice to the central significance of these topics.

But there is also much to praise in the book. It provides a consistent framework for the discussion of the enormous range of problems still in active development in the aircraft gas turbine. Chap. 15, on "Working criteria of dynamic similarity," is a welcome summary of the nomenclature which has grown up in specifying aircraft engine performance. This discussion would have had added strength if the author could have included brief references to some of the other concepts, such as velocity ratio, Parson's number, specific speed, etc., which have become established in other fields. Proper emphasis is given to the intake and exhaust systems in aircraft, even though the treatment of this important subject is far from exhaustive.

The book will find its place as a valuable addition to the growing literature on the aircraft gas turbine, and the beginner will get from it a fairly complete perspective of the range of problems involved. C. R. Soderberg, USA

3031. Strscheletzky, M., Gap losses in axial flow machinery, especially in Kaplan turbines (in German), *Forsch. Geb. Ing.-Wes.* **21,** 4, 101–106, 1955.

Paper is a contribution to the improvement of efficiency of axial flow runners through hydrodynamic analysis of the flow in the clearance between casing and blade edge. The effects of the boundary layer in which the blade edge moves and the radial flow of boundary layer on the surface of the blade are considered. A formula for the loss of energy is given. Some experimental data concerning clearance losses in a rotating disk are given in function of peripheral velocity, Reynolds number of clearance, disk thickness, and clearance width. Author indicates how to use this coefficient to calculate coefficient for axial flow machines. Circulation and free vortex systems are analyzed and "induced" or "clearance" efficiency ("Spaltwirkungsgrad") is defined. Similarity considerations concerning clearance flow are made and a law is derived so as to have the same ratio of clearance width/boundary-layer thickness in both model and prototype. Paper closes with main results and conclusions from theory and experiments. Under the best conditions and for current machinery, clearance losses amount 1% in turbines and 3% in turbines.

E. O. Macagno, Argentina

3032. Zysina-Molozhen, L. M., Approximate method for calculating losses in cascade profiles of turbomachines (in Russian), *Teploenergetika* **2**, 9, 43–48, 1955.

Research on the outer part of the boundary layer leads to modification of L. H. Loitsianskii's method of calculating boundary-layer losses. The new method enables calculation of losses in cascade profiles for wider range of angle of attack.

Comparative graphs of calculation and experimental results are shown.

W. B. Palijenko, Canada

3033. Hamrick, J. T., Some aerodynamic investigations in centrifugal impellers, *Trans. ASME* **78**, 3, 591–602, Apr. 1956.

Theoretical and experimental investigations on flow through rotating impeller passages are presented. Scope of paper is to acquaint designer with the manner in which losses arise, their effect upon the flow, and the over-all effect upon impeller performance. Also, analysis and design methods based on isentropic-flow calculations are discussed and their application to the design of mixed-flow impellers is demonstrated.

From author's summary by A. Balloffet, USA

3034. Helms, H. E., Finding the external support reactions of an Allison turbo-prop engine, *GM Engng. J.* **3**, 1, 16–21, Jan. Feb. 1956.

3035. Bowerman, R. D., The design of axial flow pumps, ASME Ann. Meet., Chicago, Ill., Nov. 1955. Pap. 55-A-127, 22 pp. + 20 figs.

Design procedures are developed for propeller pumps based on the general methods of Spannhake with corrections for three-dimensional mutual-interference effects. Design performance is compared with experimental observation. Induced interference velocities at impeller blades are calculated, including interference of the other blades and of the total downstream vorticity along the center line of the pump. A complete series of tests on one two-bladed propeller of nominal specific speed 10,000 was completed in the California Institute of Technology Hydrodynamics Laboratory rotating channel equipment. Experimental results for off-design performance are included. Comparison of the new design procedure, experimental results, and two cascade-theory design methods illustrate the value of the new design procedure. Cavitation susceptibility is thought to be reduced by designing all radial sections as equally susceptible to cavitation.

Reviewer believes the material presented does not fully substantiate the assertion regarding cavitation performance.

R. G. Folsom, USA

3036. Sabatiuk, A., and Sisto, F., A survey of aerodynamic excitation problems in turbomachines, *Trans. ASME* **78**, 3, 555–564, Apr. 1956.

Authors have succeeded in presenting a concise and most interesting summary of blade-vibration problems. Not only do they outline the present state of development, equally stimulating to the general turbine engineer and the specialist in the field, but they also point out possible and necessary future research in this field, where it appears, despite an ever increasing number of contributions, that much basic work remains to be done.

J. R. Schnittger, Sweden

3037. Lemmerman, R. D., and Lockwood, H. J., Test cell augmenter design, *Aero. Engng. Rev.* **14**, 3, 37–43, Mar. 1955.

3038. Baxter, A. D., The prospects and problems of rocket propulsion for aircraft, *J. roy. aero. Soc.* **59**, 533, 315–338, May 1955.

Flow and Flight Test Techniques

(See also Revs. 2953, 2970, 2988, 2999, 3054, 3078, 3128, 3134, 3151)

3039. Rivas, M. A., Jr., Shapiro, A. H., On the theory of discharge coefficients for rounded-entrance flowmeters and venturis, *Trans. ASME* **78**, 3, 489-497, Apr. 1956.

The contraction section of the nozzle is replaced by an equivalent frictional length L_{eq}, meaning that fictitious length of a straight tube of diameter D, which, if placed before the actual length of cylindrical tube of length L (up to the plane of downstream static taps), would lead to a discharge coefficient identical with the discharge coefficient produced by the actual combination of bellmouth and cylindrical section. Flow is assumed to enter the tube of length $L' = L_{eq} + L$ with uniform velocity and no boundary layer. Calculations are restricted to laminar boundary-layer flow. The value of L_{eq}/D is computed by an iterative procedure, to determine the boundary-layer growth, involving an initial calculation of the pressure distribution of the potential flow. The theoretical curves of discharge coefficient versus diameter Reynolds number are in good agreement with experimental data over a range of Reynolds number from 1 to 10^6.

E. Eujen, Germany

3040. Schlag, A., The classical venturi tube: Recent experiments on the effect of roughness of the converging section and of the duct in which the tube is inserted (in French) *Chaleur et Industrie* **37**, 369, 77–81, Apr. 1956.

Paper is summary of tests made with liquid to determine effect of dirty converging section and duct roughness on coefficient of discharge. In general, duct roughness increases coefficient slightly; dirt in converging section decreases it. Numerical results, presented in charts and tables, should be of interest to users of venturi tubes for accurate liquid measurements.

C. W. Smith, USA

3041. Bell, K. J., and Bergelin, O. P., Flow through annular orifices, ASME Spring Meet., Portland, Ore., Mar. 1956. Pap. 56-S-22, 18 pp. + 10 figs.

The annular orifice-type clearances which are present in some heat exchangers between the shell and the baffles and the tubes and the baffles cause a decrease in the pressure drop and in the heat-transfer coefficient. Hence it is important to know the pressure-loss coefficients of such orifices. Authors studied the coefficients of several annular orifices as a function of length-to-width ratios (0.118 to 33.3) and of orifice Reynolds numbers (5.0 to 20,000). The annular orifices which were studied differed in diameter ratios (0.95 to 0.996), in edge shapes (sharp, square, and

round), and in the manner they were mounted (concentric to form ring and eccentric to form crescent orifices). Authors used an analysis which is based partly on theoretical and partly on empirical methods. They also present experimental data. The equations which were developed by the authors permit the prediction of pressure-loss coefficients for annular orifices within a few per cent, as is demonstrated by comparisons with the experimental data. R. N. Weltmann, USA

3042. Roshko, A., Some measurements of flow in a rectangular cutout, *NACA TN* 3488, 21 pp., Aug. 1955.

The flow in a rectangular cavity, or slot, in the floor of a wind tunnel is described by the results of pressure and velocity measurements. Pressure distributions on the cavity walls as well as measurements of friction are presented. The effects of varying depth-breadth ratio are shown. From author's summary

3043. Fraser, D., and Baxter, D. C., Reference pressure probes for an orifice-type icing detector, *Nat. aero. Establish. Canad., Note 12*, 16 pp. + 19 figs., 1956.

The operation of an orifice-type icing detector depends on having a suitable reference pressure source. A flush-type static vent may not provide this, especially when the detector is used in an engine intake duct. However, tests show that a production form of icing-detector probe can be modified to provide a suitable reference pressure if it is de-iced or anti-iced. A reference probe which is particularly insensitive to yaw is also described.
From authors' summary

3044. Rebuffet, P., Some strain gage balances used in French wind tunnels (in French), *ONERA NT* 31, 57 pp. + 30 figs. + 29 plates + 1 table, Jan. 1956.

Paper was prepared for AGARD and presented in both the original French and in its English translation. It provides a most useful review of French practice for measuring steady-state components of wind loading, but the dynamic balances used for unsteady phenomena, and for measuring aerodynamic derivatives, are specifically excluded from the review. A brief general description of the principles involved in the design of various types of balance systems, and of the methods used for compensating for thermal effects and for mechanical interaction between the components, is followed by detailed descriptions of a number of balances now in use. The latter, perhaps, provides the main interest for other workers in this field, since information on the range, sensitivity, and accuracy of the balances is included.
C. Scruton, England

3045. Holmes, L. N., and Haines, A. B., High-speed wind-tunnel tests on six wings of 40-deg sweepback, with various section shapes, *Aero. Res. Counc. Lond. Rep. Mem.* 2930, 39 pp., 1955.

3046. Hollyer, R. N., Jr., Attenuation in the shock tube: I. Laminar flow, *J. appl. Phys.* **27**, 3, 254–261, Mar. 1956.

By employing a coordinate system fixed to a plane shock front, the nonstationary boundary layer in a shock tube is reduced to the stationary case. The classical boundary-layer equations are solved with the boundary condition that the wall is moving. The solution, subject to the usual assumptions of one-dimensional flow, is applied to the region between the shock and the contact front in a shock tube in order to determine the effects of heat conduction, work done on the gas by the wall, and skin friction on shock-wave attenuation with distance.

Although the present solution of the nonstationary laminar boundary-layer equations is a commendable piece of work, reviewer believes that its subsequent application to the study of shock-wave attenuation is doubtful for the following reasons:

(1) Shock-wave attenuation with distance is caused by the boundary-layer development in the entire flow, from the head of the rarefaction wave to the shock front. Consequently, the region between the shock front and the contact front cannot be treated in isolation, because it is affected by the flow between the head of the rarefaction wave and the contact surface. The latter flow occupies the major portion of shock-tube flows, and in the limit the cold flow region is sixfold greater than the hot region for infinite shock strength (constant specific heats).

(2) It is far from established that in a shock tube a laminar layer exists except immediately behind the shock front.

The best approach to shock-wave attenuation with distance, to date, has been given by Trimpi and Cohen [AMR **8**, Rev. 3508]. Their predictions have been well substantiated by several experimentors, and from all of these results it appears that the boundary layer is turbulent. A critical review of shock-wave attenuation is given in "Effects on kinematic viscosity and wave speed on shock wave attenuation," by D. W. Boyer *UTIA tech. Note* 8, to be published. I. I. Glass, Canada

3047. Tamaki, F., A divergent shock tube for obtaining supersonic flows, *J. phys. Soc. Japan* **11**, 4, 434–439, Apr. 1956.

A small shock tube having a divergent low-pressure chamber of 1×6 cm^2 initial cross section at the partition diaphragm is presented as a simple method for obtaining supersonic flows of fairly high Mach numbers. Flow character in the divergent channel is examined both theoretically and experimentally. Interferometric studies of the flow past a double-wedge profile are presented for Mach numbers of 4.0 and 6.0 and Reynolds numbers based on chord length of 6.6×10^5 adn 4.7×10^5, respectively. At $M = 6.0$, pressure distribution on the wedge surface deviates considerably from theory due to the thickening of the boundary layers.
From author's summary by W. Wuest, Germany

3048. Chul-Soo, K., Experimental studies of supersonic flow past a circular cylinder, *J. phys. Soc. Japan* **11**, 4, 439–445, Apr. 1956.

Experimental studies using shock tubes are presented of the mixed flow around a circular cylinder placed normal to the general flow of Mach number 1.35, 1.8, 2.7, 3.5, 4.0, and 6.0. With the Mach-Zehnder interferometer and schlieren method the following were measured: (a) shape of detached shock wave; (b) distance of shock wave from the cylinder nose; (c) pressure coefficient on the surface of the cylinder; (d) density distribution behind the detached shock; (e) streamlines in subsonic region.
From author's summary by W. Wuest, Germany

3049. Merle, M., Experimental study of gas flow (in French), *Publ. sci. tech. Min. Air, France* 308, 90 pp., 1956.

Author describes shadowgraph, schlieren, and interferometer techniques for optical study of flowing gas systems. The use of a beam of ultrasonic energy traversing the system normal to the gas flow for the measurement of gas velocity and temperature is described. Plane ultrasonic waves are introduced by an electrically driven transducer, at sufficiently high intensity to produce visible shading of their pressure maxima when observed in the schlieren system. The spatial displacement of the waves under the flow condition is related to the flow velocity and temperature, which are thus measured without placing obstructions in the flow. Photographs of results and a bibliography are included.
J. S. Arnold, USA

3050. Gracey, W., Wind-tunnel investigation of a number of total-pressure tubes at high angles of attack—subsonic, transonic, and supersonic speeds, *NACA TN* 3641, 30 pp., May 1956.

The total-pressure errors of 54 total-pressure tubes at angles of attack up to 60° and Mach numbers from 0.26 to 1.62 are presented. The configurations of the various tubes differed as regards external shape, internal shape, and type of total-pressure entry. External shapes tested included cylindrical, conical, and ogival nose sections. Internal shapes and total-pressure entries

were varied on the basis of such factors as size of impact openings, internal chamber shape, leading-edge slant profile, and shielding (combined with venting). The effect of these variables on the sensitivities of the tubes to angle of attack is presented. [See AMR **4**, Revs. 1763 and 4596]

From author's summary

3051. Lovell, P. M., Jr., and Parlett, L. P., Hovering-flight tests of a model of a transport vertical-take-off airplane with tilting wing and propellers, *NACA TN* 3630, 23 pp., Mar. 1956.

An investigation of the take-off, landing, and hovering-flight characteristics of a four-engine transport vertical-take-off airplane with tilting wing and propellers has been conducted with a remotely controlled free-flight model. The model had four propellers distributed along the wing with the thrust axes in the wing-chord plane. In order to produce direct lift for hovering flight with the fuselage horizontal, the wing and propellers were rotated 90° with respect to the fuselage. Despite the fact that the pitching and rolling motions of the model were unstable oscillations, the model could be flown smoothly and easily without the use of any automatic stabilization devices because the periods of the oscillations were fairly long and the controls were powerful. The pitching oscillation could be completely stabilized by the use of artificial damping in pitch; thus the model could be flown in pitch for long periods of time without the use of the manual pitch control. Although there was no stability of yaw position, the model was easy to control in yaw because the motions were slow and the yaw control was powerful. There were no noticeable interactions between the rolling and yawing motions or between the roll and yaw controls. Vertical take-offs and landings could be performed fairly easily, although some forward or backward motion of the model was often present.

From authors' summary

Thermodynamics

(See also Revs. 2940, 2959, 2974, 3023, 3030, 3038, 3041, 3071, 3075, 3078, 3080, 3085, 3116)

3052. Shanks, D., A study of postulates: the "thermodynamic" derivation of the adiabatic gas law, *Amer. J. Phys.* **24,** 5, 352-354, May 1956.

Author shows that the reversible adiabatic law can be derived from the definitions of the adiabatics and of the specific heats without involving the first law or the thermodynamic temperature scale.

Reviewer agrees with author's derivation but feels that too little emphasis has been placed on the condition of reversibility, actually a second-law concept. Without this restriction, author's definitions of adiabatic change and of specific heats hardly have any physical sense, for then the heat flow along given paths is no more unique.

L. S. Dzung, Switzerland

3053. Popoff, K., Thermodynamics of irreversible processes (in French), *ZAMP* **6,** 5, 378-386, 1955.

3054. Chuan, R. L., On the method of averaging stagnation pressures, *J. aero. Sci.* **23,** 4, 378-379 (Readers' Forum), Apr. 1956.

In averaging total pressures, the maximum value of the average is obtained by taking the total pressure that corresponds to the mass average of the entropies. Comparison is made with the mass average of total pressure. [NOTE: There is a typographical error in Eq. 7.]

N. Van Le, USA

3055. Zwanzig, R. W., Virial coefficients of parallel square and parallel cube gases, *J. chem. Phys.* **24,** 4, 855-856, Apr. 1956.

The second, third, fourth, and fifth virial coefficients are evaluated exactly for gases whose molecules interact in two dimensions as rigid elastic squares with their sides parallel or in three dimensions as rigid elastic cubes with their edges parallel. Upper and lower bounds on the virial coefficients of gases composed of rigid circles or rigid spheres are obtained in terms of the virial coefficients for the parallel squares and parallel cubes.

From the author's summary by J. S. Marcus, USA

3056. Christian, R. H., and Yarger, F. L., Equation of state of gases by shock wave measurements. I. Experimental method and the Hugoniot of argon, *J. chem. Phys.* **23,** 11, 2042-2044, Nov. 1955.

An experimental method is described for the simultaneous measurement of the velocity of a plane shock through a gas and the associated particle velocity. Shocks are generated by a plate driven by a high-explosive system. The velocities are recorded by a high-speed rotating-mirror smear camera with a precision of about ±½%. Data are presented defining the Hugoniot of argon between 200 and 1100 Los Alamos atmospheres. These data are compared with the theoretically predicted Hugoniot.

From authors' summary

3057. Christian, R. H., Duff, R. E., and Yarger, F. L., Equation of state of gases by shock wave measurements. II. The dissociation energy of nitrogen, *J. chem. Phys.* **23,** 11, 2045-2049, Nov. 1955.

The results of equation-of-state measurements made behind strong shock waves in nitrogen are consistent only with the higher of the two spectroscopically acceptable values of the dissociation energy of nitrogen, 9.764 ev.

From authors' summary

3058. Hirschfelder, J. O., Buehler, R. J., McGee, H. A., Jr., and Sutton, J. R., A generalized equation of state for both gases and liquids, *Univ. Wisc. Nav. Res. Lab. Rep.* Wis-OOR-12 (9 & 10), 47 pp., Mar. 1956.

Title equation is derived by a semiempirical method, based on a generalized principle of corresponding states. The equation for gas gives the pressure as a function of temperature and density when all or some of the four parameters—critical pressure, critical temperature, critical density, normal boiling point—are known. Equation for liquid involves knowledge of saturated liquid density at any two different temperatures.

Agreement with experiment values is usually acceptable for rough estimation. The equation will be valuable for substances for which very few measured data exist. For liquid water, the given equation is entirely inadequate.

The equation for high-density gas (both temperature and density greater than critical) does not satisfy the condition given by Tsien [AMR **9,** Rev. 269], which requires that at infinite temperature the gas must behave as a perfect gas even if the specific volume remains finite at a small value.

L. S. Dzung, Switzerland

3059. Jeffreys, H., The thermodynamics of thermal instability in liquids, *Quart. J. Mech. appl. Math.* **9,** 1, 1-5, Mar. 1956.

In marginal instability the supply of energy of expansion must just balance the dissipation. This principle is developed for the problem of a fluid sphere heated within and expressed in terms of the disturbance of temperature.

This provides a principle that can be used to test stability. This argument can be extended, but will not be simplified.

W. L. Sibbitt, USA

3060. Acharya, Y. V. G., The design of a cylindrical injector, *Appl. sci. Res. (A)* **5,** 4, 296-308, 1955.

The equations for a cylindrical injector have been deduced from simple momentum considerations, and a design formula, based upon those equations, is suggested. Series of experiments, specially designed to verify this formula, are described, leading to

the conclusion that the formula can be used for predicting the ratio of the velocities of the primary and the induced secondary streams if use is made of a couple of empirical coefficients whose values have been deduced from the experimental data.

From author's summary

3061. Johnston, I. H., The effect of inlet conditions on the flow in annular diffusers, *Aero. Res. Counc. Lond. curr. Pap.* 178, 8 pp. + 7 figs., 1954.

Tests have been carried out on annular diffusers having a common area ratio of 3.19 and varying in divergence angle from 6.5° to 15°. The performance of each diffuser has been measured for a variety of inlet velocity distributions and the effect of axially splitting the flow in the diffusers has been investigated.

Diffuser efficiency is found to deteriorate as inlet conditions become nonuniform, this tendency increasing with diffuser angle.

Splitting of the higher angle diffusers improves efficiency for nonuniform profiles, but these increases in efficiency are accompanied by pronounced static pressure gradients across the diffuser throat which, in certain applications, might prove undesirable.

From author's summary

3062. Smit, R., Influence of humidity on measurements with radiation pyrometers, *Appl. sci. Res. (B)* **5,** 6, 428-436, 1956.

The infrared absorption bands of water vapor at room temperature affect radiation measuring instruments whose range of sensitivity includes these bands. The corrections to target temperatures (range 600-1300 C) are shown to be 20-50 C when observed, through an atmosphere having an optical density of 6.5 cm atm of H_2O, by thermopile devices. Such corrections are therefore caused by relative humidities of 65% at 25 C, through a 3-m path length of air.

M. Gilbert, USA

Heat and Mass Transfer

(See also Revs. 2856, 2994, 3023, 3037, 3041, 3043, 3054, 3062, 3093, 3096, 3101, 3110, 3134, 3136)

3063. El-Hifni, M. A., and Chao, B. T., Measuring the thermal diffusivity of metals at elevated temperatures, *Trans. ASME* **78,** 4, 813-821, May 1956.

Author describes an arrangement of the Angström-bar method employing a "steady" sinusoidal heat flow in which due account is taken of the influence of a small heat loss at the surface of a thin-walled tubular specimen. Thermal diffusivity was calculated from the phase shift and amplitude ratio of the temperature waves determined at two measuring stations. Measurements were made for temperatures ranging from 300 to 1650 F, on AISI 1018, 1045, 3140 steels, on 1.1%-carbon tool steel, on electrolytic tough-pitch copper (purity 99.90%), on 25 aluminum (purity 99.0%). Where other data existed, agreement was good, except for a discrepancy which appears between results obtained here for 99.0% aluminum and those found for the 99.2% aluminum. Logarithm of thermal diffusivity of copper and aluminum varies linearly with temperature; this agrees with Storm's theory for simple metals (Al, Cu, Pb, Zn, Ag).

A. E. Brun, France

3064. Manson, S. S., and Smith, R. W., Quantitative evaluation of thermal-shock resistance, *Trans. ASME* **78,** 3, 533-544, Apr. 1956.

The dependence of thermal-shock resistance on two parameters k and $\sigma_f/E\alpha$ (when k is conductivity, σ_f the fracture stress, E the elastic modulus, and α the coefficient of thermal expansion) is experimentally demonstrated by the fact that the relative ratings of two materials may change with a change of quench severity. The theory of thermal-shock resistance of a disk quenched at the periphery is developed, making use of the results of Jaeger. Two criteria for failure are considered; namely, the maximum normal-stress criterion, and the criterion of maximum "risk of rupture" based on the Weibull statistical theory. Equipment for subjecting these specimens to a wide range of quench severities of known surface heat-transfer coefficients is described, and results are presented for thermal-shock tests of steatite and glass disks. The results for steatite are then utilized for determination of thermal-shock parameters by four independent methods, the results showing good agreement.

From authors' summary by E. A. Davis, USA

3065. Mendelson, A., and Manson, S. S., Approximate solution to thermal-shock problems in plates, hollow spheres, and cylinders with heat transfer at two surfaces, *Trans. ASME* **78,** 3, 545-553, Apr. 1956.

Authors present a polynomial approximation method whereby the heat-conduction equation is reduced to a set of first-order ordinary differential equations. Examples treated in appendixes are hollow cylinders, plates, and spheres. Resulting accuracies are compared with exact or analog solutions and show favorable comparison only at distances relatively far from the heat input surface, a failing which is common to most solutions to this problem. Reviewer believes that for solutions in the vicinity of sharp thermal gradients for short time periods too many stations will be needed, so that no reduction of the computational problem will result over (say) a machine-programmed exact solution.

The main difficulty with exact solutions is that they are generally in infinite series of higher transcendental functions, which, for the conditions stated, are slowly convergent [e.g., Awbery, *Phil. Mag.* **28,** p. 447, 1939]. However, the convergence of the approximate solution to the exact solution is no better off when the number of stations needed is taken into account. The same difficulty exists with mesh-type analog computers.

A. C. Block, USA

3066. Kneschke, A., Cooling and heating of solid bodies under controlled conditions (in German), *Ing.-Arch.* **24,** 2, 77-80, 1956.

The rate of heat transfered to or from a solid body may be controlled by controling the temperature of the surrounding air; in particular, by appropriately varying this temperature. The problem of calculating the corresponding distribution of temperature in the solid body is solved in the present paper by means of analytical standard methods resulting in correct, if cumbersome, expressions of great generality. The solutions offered are not complete, their evaluation depending on further mathematics (boundary-value problems). An important result however, is obtained by showing that the solution is, in full, reducible to the solution of a simpler problem, i.e., heat exchange with air of constant temperature, which may be soluble by direct measurement.

Reviewer wonders whether the author's brilliant mathematical analysis lends itself to numerical computation. It may be useful for systems, such as regenerators, where reduction to the simpler problem is practicable and where high accuracy is required.

R. Eisenschitz, England

3067. Manfredi, B., Numerical solution of one-dimensional heat conduction (in Italian), *Riv. Mat. Univ. Parma* **6,** 1-2, 141-155, 1955.

The time-dependent one-dimensional heat equation is discussed. Methods are outlined for transforming the differential equation into one of finite differences. Extension of Duhamel's theorem for discrete values of temperature distribution in space and in time is discussed. Duhamel's integral equation is given in a finite series form and convergence properties of the series are discussed. The time-dependent heat conduction in a semi-infinite slab is discussed in some detail.

S. Eskinazi, USA

3068. Datsev, A. B., On the two-dimensional multi layer problem of heat conduction (in Russian), *Dokladi Akad. Nauk SSSR*

(N.S.) **101**, 5, 813–816, 1955 (translated from Russian by M. D. Friedman, 572 California St., Newtonville, Mass., 7 pp.).

Author presents a formal mathematical treatment of the problem of two-dimensional heat conduction in two homogeneous bodies where one body is bounded by a smooth contour and the second body is the entire region exterior to the contour. It is assumed that the temperature is known initially everywhere and it is desired to find the temperature distribution as a function of time. Exploiting the technique used in previous papers, author shows that the desired solutions can be obtained through the solution of an integral equation, and outlines the method of solution. The problem is then formally extended to consider n homogeneous cylindrical bodies bounded by the smooth contours $l_1, l_2 \ldots l_N$ where each contour l_i contains the remaining $l_{i+1}, \ldots l_N$. No explicit solutions are given.

Reviewer believes that this work will be of interest to the specialist in the field. G. C. Wallick, USA

3069. Datsev, A. B., On the three-dimensional multi layer heat conduction problem (in Russian), *Dokladi Akad. Nauk SSSR (N.S.)* **101**, 6, 1019–1021, 1955 (translated from Russian by M. D. Friedman, 572 California St., Newtonville, Mass., 5 pp.).

Author presents a formal and direct extension of his method of solution of the two-dimensional heat conduction problem [See preceding review] to the three-dimensional problem. An integral equation formulation of the solution of the two-region problem is again presented. This treatment is then formally generalized to the problem of n spatial layers $R_1, \ldots R_n$ when each R_i layer contains the R_{i+1} layer. As in the two-dimensional presentation, method of solution is outlined but no explicit solutions are given. Reference to the author's previous papers would be of value to anyone interested in a clarification of the three-dimensional development. G. C. Wallick, USA

3070. Groth, K., Temperature distribution and low cylinder wall temperature of a diesel motor (in German), *Motortech. Z.* **16**, 10, 277–284, Oct. 1955.

3071. Momot, Ch., Graphical investigation of heat exchange questions in rotary cement kiln (in French). *Rev. Mater. Constr.* no. 460, 17–30, Jan. 1954.

3072. Clark, S. K., Heat-up time of wire glow plugs, *Jet Propulsion* **26**, 4, 278–279, Apr. 1956.

3073. Osipov, K. A., and Fedotov, S. G., Heat capacity and mechanical properties of metals (in Russian), *Izv. Akad. Nauk SSSR Otd. tekh. Nauk* no. 2, 98–104, Feb. 1955.

3074. Hayes, W. D., On laminar boundary layers with heat transfer, *Jet Propulsion* **26**, 4, 270–274, Apr. 1956.

The methods developed permit computation of local shear stress and heat-flux rate in laminar compressible boundary-layer flow of an imperfect gas over two-dimensional and three-dimensional axisymmetric bodies of arbitrary shape.

The two-dimensional boundary-layer equations are reduced to approximately (Prandtl number unequal to one) incompressible form for an imperfect gas by a variation of the Stewartson-Illingworth transformation. (The three-dimensional reduction by the Mangler transformation is indicated.) Employed in the reduction is an approximately valid relationship between the density and total enthalpy ratios, which satisfies one free-stream and two wall conditions. The reduced boundary-layer equations are then integrated across the boundary layer for use with an integral method that assumes similarity relationships for the momentum thickness, form factor, and shear stress at the wall applicable to the momentum equation and for the enthalpy-defect thickness and heat-flux rate at the wall applicable to the energy equation. The classical method of solution, which solves the momentum equation independently of the energy equation, leads to two different descriptions of the heat-flux rate at the wall—this inconsistency reveals somewhat of an inaccuracy in the method. Several methods of solving the momentum and energy equations, which are graded between the classical solution and the simultaneous solution of both equations, are indicated, one's choice of which would depend on expected relative accuracy and computational speed. E. V. Somers, USA

3075. Sigwart, K., and Nassenstein, H., Mechanism of mass transfer in the boundary between two liquid phases, (in German), *ZVDI* **98**, 11, 453–461, Apr. 1956.

Mass transfer is based on the two-film theory. Since actions taking place at the boundary play an important part, the transfer of a soluble substance through the boundary of two liquids has been studied by means of schlieren photographs. The experiments indicate that with many multi-component systems eruptions take place. Their intensity depends on the concentration of the solute and on the presence of interfacial-active additives (impurities). The flow pattern is determined by the interfacial tension which depends upon the concentration of the solution, as demonstrated by some model experiments.

The process of bursting and closing of a liquid film around a droplet growing at the end of a capillary with a periodic sequence of vortical rings can be interpreted as a physical interfacial phenomenon.

From authors' summary by W. Gumz, Germany

3076. Toor, H. L., The energy equation for viscous flow, *Indust. Engng. Chem.* **48**, 5, 922–926, May 1956.

Paper presents an interesting analysis of the flow of viscous fluids with thermal effects included. The energy equation in particular is developed so as to include the effects of internal generation of heat on fluid properties and temperature distribution. The behavior of non-Newtonian fluids is also considered. It is shown that, even in the case of liquids, the assumption of incompressibility may not be valid if frictional heat generation is significant.

M. J. Thompson, USA

3077. Ribaud, G., Some remarks on the subject of heat and momentum transfer in the boundary layer (in French), *C. R. Acad. Sci. Paris* **240**, 1, 25–28, Jan. 1955.

Author uses what amounts to the integral form of the momentum and energy equations for incompressible boundary-layer flow to derive a specification of the local heat-transfer coefficient for constant wall temperature. The laminar velocity profile is represented by a quadratic and the turbulent profile by a one-seventh power law. The Prandtl number is assumed equal to unity.

D. L. Doughty, USA

3078. Bryant, L. W., and Cowdrey, C. F., Effects of velocity and temperature of discharge on the shape of smoke plumes from a funnel or chimney: experiments in a wind tunnel, *Instn. mech. Engrs. Proc.* **169**, 23, 371–400, 1955.

From a limited number of model observations of hot and cold plumes, extrapolation to practical conditions is made in order to estimate especially the effect of washing on the reduction of sulphur-dioxide concentration. F. R. Hama, USA

3079. Saxena, S. C., Thermal diffusion of gas mixtures and determination of force constants, *Indian J. Phys.* **38**, 3, 131–140, Mar. 1955.

The value for the force constant ϵ_{12} has been calculated from the observed variation of thermal diffusion factor with temperature to a higher approximation than before. These ϵ_{12} values have been combined with binary viscosity and interdiffusion data to calculate r_{12}. Utilizing these values of the force constants the thermal diffusion factor has been calculated for a number of binary gas mixtures and compared with the experimental data. It is found

that the agreement between theory and experiment is quite satisfactory. From author's summary

3080. Meyer, R. E., and Nachtrieb, N. H., Self-diffusion of liquid sodium, *J. chem. Phys.* **23,** 10, 1851–1854, Oct. 1955.

The self-diffusion of liquid sodium in the temperature range 98 C to 226 C has been studied by the capillary reservoir technique. The data fit the empirical equation, $D = 1.10 \times 10^{-3} \exp(-2430/RT)$. The Stokes-Einstein equation gives a good correlation of the self-diffusion and viscosity data. The difference in the self-diffusion coefficients of the solid and liquid sodium is explained in terms of the "relaxation" theory of diffusion proposed by Nachtrieb and Handler.

From authors' summary

3081. Eckert, E. R. G., and Irvine, T. F., Selection of optimum configurations for a heat exchanger with one dominating film resistance, ASME Spring Meet., Portland, Ore., Mar. 1956. Pap. 56-S-20, 20 pp. + 16 figs.

An "optimization" (horrid word!) is carried out to determine which of seven heat-exchanger geometries gives the smallest energy loss, heat-exchanger weight, volume or frontal area, it being assumed that the heat resistance on one side of the exchanger is so small that it can be neglected. A study is also made of the effect of arranging the exchanger at an angle with the oncoming coolant streams, so giving a larger frontal area to the exchanger for a constant duct diameter. It is shown that the power consumption can vary five to eight times in the examples given for equivalent weight and heat flow.

G. G. Thurlow, England

3082. Fournel, E., Means for improving efficiency of tubular heat exchangers (in French), *Chaleur et Industrie* **37,** 368, 53–58, Mar. 1956.

Wire of sinuous form, wound on the surface of a tube, increases the heat-transfer coefficient α by 15 to 110%, according to velocities of air cross-flow varying from 6 to 50 m/s. Relation between numbers of Margoulis ($=Nu/Pe = \alpha/Gc_p$) and Reynolds ($= Gd/\mu$) for different tube- (10 to 28.5 mm) and wire-diameters (0.5 to 1.5 mm) is quoted, but general formula is not derived. By altering the distance of wire-windings along the tube it is possible to change α in the way prescribed. Different ways of fastening the copper wire on the tube are discussed.

O. Mastovsky, Czechoslovakia

3083. Eckert, E. R. G., and Diaguila, A. J., Method of calculating core dimensions of crossflow heat exchanger with prescribed gas flows and inlet and exit states, *NACA TN* 3655, 25 pp., Apr. 1956.

A method is presented for calculating the necessary dimensions of a heat exchanger to give a specified performance; the entry conditions, flow rates, temperature and pressure differentials, and type of heat-transfer surface being fixed. The three linear dimensions together with the Reynolds numbers (as a criterion of velocities) of the two fluids give five unknowns, requiring five equations to be solved.

Paper develops a method of setting up and solving this number of equations, necessitating a "trial-and-error" procedure. The procedure is simplified by the use of graphs, making it a fairly simple matter to examine a heat-exchanger problem over a range of conditions.

Method will be of great practical value to people engaged in heat-exchanger design, especially where unit size is a major consideration. G. G. Thurlow, England

3084. Nagai, M., Graphical solution of linear heat flow with radiation, *J. phys. Sec. Japan* **11,** 3, 329–330 (Short Notes), Mar. 1956.

The second-order nonlinear differential equation representing heat conduction along, and uniform heat generation in, a wire with fixed end temperatures and radiation from its surface at a local rate proportional to the fourth power of the absolute temperature is solved by a standard mathematical technique. The results are represented graphically in nondimensional coordinates.

R. L. Pigford, USA

3085. Tabor, H., Solar energy collector design with special reference to selective radiation, *Bull. Res. Counc. Israel* **5(C),** 1, 5–27, Dec. 1955.

Paper analyzes heat-transfer processes within solar collectors, giving particular attention to reducing the outward heat loss by replacing conventional black-painted absorbing surface with a special surface that absorbs more than 90% of the incident solar radiation yet has an emissivity of less than 10%. [See H. Tabor, title source **5A,** 119–134, Jan. 1956.] The concept of heat-retention efficiency is introduced and generalized yearly average efficiency curves are produced. The latter are based on a typical daily radiation curve rather than on hourly radiation values such as are required for accurate computation when clouds are prevalent. Approximate methods are given for correcting for differences in solar intensity from place to place, for the effect of angle of incidence, for shading and dirt, and for fluid flow rate, and collector plate design. Typical examples are given both for heating and for power generation, which show the effect of using one or two glass plates, of using selective-radiation surfaces, and of concentrating the incoming radiation.

Paper is a valuable addition to solar energy literature. It should be studied in conjunction with a similar paper, "Evaluation of flat-plate solar collector performance" by H. C. Hottel and A. Whillier [Trans., Conference on Solar Energy—The Scientific Basis, Tucson, Arizona, 1955], which introduces several similar concepts and further refining techniques. A. Whillier, S. Africa

Combustion

(See also Rev. 3037)

3086. Clarke, J. S., A review of some combustion problems associated with the aero gas turbine, *J. roy. aero. Soc.* **60,** 544, 221–240, Apr. 1956.

Paper summarizes effects of operating conditions, annular chamber type, fuel injection location, temperature traverse, pressure loss, fuel properties, and carbon formation upon combustion. Problems in flame stabilization and heat release rate are discussed. Author reports also upon progress and trends in turbine combustion chamber design, using specific examples. Materials for flame tubes and problems of ignition are also covered.

Reviewer believes the paper presents, within a few pages, a reasonably thorough discussion of turbine combustion problems. Such an article is not only of value to practicing designers who will profit from the review of problems and considerations involved in improving combustion, but also to neophytes who desire orientation. M. Popovich, USA

3087. Fristrom, R. M., Flame zone studies. II. Applicability of one-dimensional models to three-dimensional laminar Bunsen flame fronts, *J. chem. Phys.* **24,** 4, 888–894, Apr. 1956.

Variation of flow velocity along streamlines passing through stabilized laminar premixed flames has been carefully measured by intermittent illumination of entrained solid particles. Conical, button, and inverted-vee shapes of lean propane-air flames have been investigated. The concept that burning velocity depends only on initial state and is independent of flame geometry is verified for all cases studied. It is claimed that previous confusion on this point has stemmed from lack of knowledge of the flow pat-

terms. It is concluded that "the properties of the flame front are functions only of the coordinate normal to the flame front." However, the statement is also made that "one-dimensional flame theories should be modified to include the effects of area expansion (sideways) during the course of the reaction." Another important conclusion is that reactions in the secondary combustion (post-luminous) region must be coupled only loosely with the burning-velocity-determining primary reactions.

Reviewer intuitively agrees with this view; however, a theoretical investigation of the coupling in a two-stage flame seems worthwhile. R. Friedman, USA

3088. Speisher, V. A., Combustion stability of low heat yielding gases in tunnel-type jets (in Russian), *Izv. Akad. Nauk SSSR Otd. tekh. Nauk* no. 11, 129–133, Nov. 1954.

Paper is an extension of previous work [AMR **8**, Rev. 3223] to include low calorific value gases. City gas was diluted with various amounts of inerts, and blow-off limits from a 48-mm tunnel burner were determined for different air/gas ratios. The caloric value of the gas was reduced from 7600 to 1700 cal/m³ with a corresponding reduction in normal burning velocity from 45 to 10 cm/sec. Stability limits of underground gasification gas (*C.V.* ~ 900 cal/m³) on industrial tunnel burners of diameters 400 and 550 mm were also determined. No new theory is presented.

J. K. Kilham, England

3089. Lees, L., Fluid-mechanical aspects of flame stabilization, *Jet Propulsion* **24**, 4, 234–236, July/Aug. 1954.

3090. Cornell, W. G., The flow in a vee gutter cascade, *Trans. ASME* **78**, 3, 573–580, Apr. 1956.

A theory is given for the pressure drop, wake contour, and drag for vee gutters in two-dimensional, unstaggered array. The assumptions of incompressible, steady, potential flow are made with a stagnant wake bounded by free streamlines. Curves for a wide range of angles and blockage ratios are included. Although the simplifying assumptions are severe, good correlation is obtained with experimental values measured at low approach velocities.

Reviewer feels this work is of value to designers in ramjet and afterburner fields. R. H. Eustis, USA

3091. Maccallum, N. R. L., Flame blow-off from rectangular burners, *Fuel* **35**, 2, 169–177, Apr. 1956.

In burners having circular flame ports, it has been shown that at atmospheric pressure the theoretically computed velocity gradient at the wall can be used to correlate blowoff and flashback data as a function of composition. Author finds the relation also holds for blowoff of laminar butane flames from rectangular ports 0.3 cm in width and 0.2 to 5.1 cm in length. Initial lift at the center of the flame correlates with circular port data in the literature when the theoretical velocity gradient at the center of the long side of the port is used. Final blowoff correlates, but not as well, when an average gradient is used. This relation also was found to hold into the turbulent regime.

Since it is easier to relate an average gradient to a measured pressure drop than to compute the gradient theoretically, when the shape of the port is complex, pressure drop through the ports was also determined. The friction factor in the laminar flow region determined from these measurements varied in the normally expectant manner, with the inverse of the Reynolds number. From a design standpoint, this is a simpler relation than the previously reported results on somewhat larger rectangular burners [AMR **7**, Rev. 2654]. Furthermore, author's relation does not predict an approach to zero blowoff velocity as the length of burner increases, as the previous relation did. A. A. Putnam, USA

Book—3092. Frank-Kamenetskiĭ, D. A., Diffusion and heat exchange in chemical kinetics (translated from Russian ed. by N. Thon), Princeton, Princeton Univ. Press, 1955, xii + 370 pp. $6.

This volume provides an excellent coverage of the pertinent fundamentals of diffusion, heat transfer, fluid dynamics, and ideal chemical kinetics, and of their interrelationships in the field of actual or macroscopic chemical kinetics.

Chapter 1 defines terms and describes convective heat and mass transfer in a concise and authoritative way. Chapter 2 analyzes diffusion-controlled heterogeneous reactions, using the combustion of carbon and dissolution of metals as illustrations. Chapter 3 covers net or Stephan flow and its effect on diffusion rate, and chap. 4, thermal diffusion and simultaneous diffusion and heat conduction. Chapter 5 describes boundary layers at stationary and suspended solid surfaces, and the use of generalized turbulent velocity distributions and of eddy diffusivity. Chapter 6 outlines thermal ignition, explosion, and flame-propagation theory. Chapter 7 calculates temperature distributions in reaction vessels with cold walls, with generalized solutions for spherical, cylindrical, and large flat vessels. Chapter 8 covers thermal and chain propagation of flames. Chapter 9 studies the thermal regime of heterogeneous exothermic reactions, in which heat transfer and diffusion are simultaneously necessary. The theory of ignition and extinction is covered, including the effect of thermal diffusion and net flow. Chapter 10 develops the necessary conditions for periodic or oscillatory processes, such as observed in the oxidation of hydrocarbons.

This volume is suitable as a graduate text and as a background and reference book, particularly on Russian literature. The English and typography are also excellent.

C. F. Bonilla, USA

3093. Altman, D., and Wise, H., Effect of chemical reactions in the boundary layer on convective heat transfer, *Jet Propulsion* **26**, 4, 256–258, 269, Apr. 1956.

An analysis is made of the relative change in heat-transfer rate resulting from chemical reactions in the boundary layer for either turbulent or laminar flow. Authors solve simultaneously the boundary-layer equations for conservation of heat and mass, including terms describing the heat of reaction of diffusing species and the molar reaction rates, respectively. The resulting expression for relative increase of heat-transfer rate over that without chemical reaction is too general to be solved analytically. Fortunately, the simplest cases amenable to analytical treatment are those of practical importance. Assuming von Karman's analogy between fluid friction and heat transfer, the equality of the coefficients of turbulent exchange for viscosity, diffusivity, and conductivity, the constancy of Prandtl and Schmidt numbers, steady-state operation, and maintaining cold-wall temperature fixed at low value by coolant flow, authors obtain analytical solutions for three cases: (1) $D = K$ with reaction occurring anywhere in the entire boundary layer, or reaction occurring only in the hot portion of the intermediate layer where D and K are much less than E; (2) reaction restricted to surface; (3) reaction restricted to laminar layer. In order to perform integrations in cases (2) and (3), authors have to refer to auxiliary relations from the kinetic theory of gases. Paper concludes with the application of derived equations to a specific flame system transfering heat to a cold wall. In the system, liquid ammonia reacts with gaseous oxygen at a pressure of 20 atmos. The total increase in calculated heat-transfer rate amounts to 20.6 %. B. Zarwyn, USA

3094. Gaydon, A. G., and Moore, N. P. W., Spectra of cool flames and pre-ignition glows, *Proc. roy. Soc. Lond. (A)* **233**, 1193, 184–194, Dec. 1955.

Techniques for study of cool flames were extended in particular to preignition glow, which appears before ignition in the lower pressure and higher temperature. Flow system and quartz spectrograph were used.

With propane-oxygen, preignition glow in weak mixtures is like hot flame with strong OH, some HCO, and some OH. In strong mixtures appear some HCO and formaldehyde bands, CH and only weak

C_2. Normal flame shows strong C_2 and CH bands, some OH and weak HCO, whereas cool flames show formaldehyde bands. The evidence on reaction mechanisms indicates that these involve chemiluminescence rather than thermal excitation, and that, in general, C_2 and CH are associated with chain-branching and propagation at relatively high temperatures (as in flame propagation) and HCO with chain initiation and termination at relatively low temperatures (as in ignition).

No preignition glow was observed with methane, and rich carbon monoxide-oxygen mixtures showed the usual CO-flame bands. Addition of methane to lean carbon monoxide-oxygen mixtures gives spectrum similar to cool-flame bands of formaldehyde. With rich mixtures, glow is suppressed.

With formaldehyde added to carbon monoxide only, the CO-flame bands were observed. No glow was observed with formaldehyde alone at atmospheric pressure.

Significance of these results in connection with ignition in engines is discussed. R. C. Anderson, USA

3095. Kopytoff, V., Bell, C. G., and Cambel, A. B., Study of premixed flames using radioactive tracers, *Jet Propulsion* **26**, 6, 501–502 (Technical Notes), June 1956.

3096. Godel, M. A., A new technique in combustion (in French), *Mem. Soc. Ing. civ. Fr.* **108**, 6, 476–492, Nov.-Dec. 1955.

Fluidized bed operation is applied to the industrial combustion of coal. A suitable furnace and its operation are described in detail. Author believes that combustion of coal in a fluidized bed will be widely applied, since it does not require special high-ranking sorts of coal, or expensive preparation.

H. Behrens, Germany

3097. Crocco, L., Grey, J., and Matthews, G. B., Measurements of the combustion time lag in a liquid bipropellant rocket motor, *Jet Propulsion* **26**, 1, 20–25, Jan. 1956.

Sinusoidal pressure modulations were imposed on inlet propellant lines to a small bipropellant rocket engine. Amplitude and phase of resulting pressure vibrations in combustion chamber were measured. Data will be useful in developing satisfactory understanding of high-frequency pressure oscillations in rocket engines.

R. Carnog, USA

3098. Wise, H., and Agoston, G. A., The burning of a liquid droplet, Spring Meet., Amer. Chem. Soc., Dallas, Tex., Apr. 1956. 32 pp. + 8 tables + 2 figs.

Injection of reactant sprays into combustion chambers makes it necessary to study the processes governing the combustion of liquid aerosols. The theoretical and experimental aspects of burning of a single droplet are presented, which form the basic problem in combustion research. Two cases of droplet combustion are of interest: (1) a heterogeneous monopropellant flame involves a single component evaporating from the liquid surface and then decomposes exothermically at a rate dependent on temperature and composition; (2) a bi-propellant flame in which two reactants, fuel and oxidizer, interact, one of which evaporates from the droplet surface and diffuses into the gas phase containing the other reactant, as a result of which the flame is established at some distance from the droplet.

Chapters: Heat transfer by conduction; Free and forced convection: Dynamics of burning droplets; Experimental techniques (stationary, nonsteady-state combustion; stationary, steady-state combustion; nonstationary, nonsteady-state combustion); Experimental results (phases of fuel droplet: preignition period, liquid-combustion period, effect of gas temperature on burning rate, effect on burning rate of pressure, of gas-phase composition, of liquid-phase composition, and of ambient gas flow); Flame extinction from upstream portion of a drop in motion; Drag coefficient of burning drops; Application of burning-rate measurements.

Tables: Parameters for fuel-oxygen system; Preheat time preceding drop ignition; Evaporation constants; Flame-distance (natural convection); Pressure exponent; Effect of forced convection on burning rate; Drag coefficient of burning spheres.

K. J. De Juhasz, Germany

3099. Bolt, J. A., and Boyle, T. A., The combustion of liquid fuel spray, *Trans. ASME* **78**, 3, 609–615, Apr. 1956.

Burning of liquid droplets of uniform particle size was observed photographically. Particle diameter decreased with time according to equation: $D^2 = D_0^2 - \lambda t$, where D is drop diameter at time t, D_0 is diameter at t equal zero, λ is constant, cm^2/sec. Value of λ, using D in cm and t in sec for particles initially about 0.01 cm, is 0.0047 for n-heptane, 0.0046 for n-propyl alcohol, 0.0033 for benzene, and 0.0066 for cyclohexane.

Importance of study lies in measuring rates with many drops, thus simulating combustion-chamber conditions. Value of λ is about one half that obtained by others for larger single droplets suspended on fiber.

Whirling disk was used by authors to obtain the desired uniformity of drop size. Uniformity may be helpful for study and analysis, but reviewer feels some combustion systems perform better with wide spectrum of fuel drop size, as obtains with air atomization. As particles undergo combustion even with initial uniformity, there is an increasing spread of droplet size.

Reviewer agrees with authors' conclusion that problem of combustion is complex. D. Aronson, USA

3100. Walker, V., Distribution of insoluble additive particles in a fuel spray, *Fuel* **35**, 2, 153–160, Apr. 1956.

Given a mixture of solid, insoluble additive particles in a fuel oil, the object of the investigation was to determine to what extent the ratio of additive to fuel in the spray would vary from the over-all ratio in the mixture. Using known data about the distribution of particle sizes in the additive powder and the distribution of fuel droplet sizes in a spray, together with an assumption concerning the nature of the fuel-additive mixture, the problem was solved by elementary statistical reasoning. A set of curves has been constructed by means of which suitable combinations of overall additive to fuel ratio, and additive powder and fuel spray finenesses can be selected. Chapters: Concept of mixture; Particle distribution functions; Distribution of additive particles in the spray; Results presented in a family of curves expressing the evenness of additive distribution in the spray as ordinate, versus the proportion of additive to fuel as abscissa.

K. J. De Juhasz, Germany

3101. D'iakonov, G. K., and Semenov, G. A., Heat exchange in a layer of spheres under steady state of flow (in Russian), *Izv. Akad. Nauk SSSR Otd. tekh. Nauk* no. 7, 109–118, July 1955.

Authors report on experimental research on the heat exchange between a layer of spherical bodies and a gas flowing through it with the spheres being warmer than the gas, representing the simplified case of practical occurrences such as reaction columns with granular catalysts, heat regenerators, stokers with thick fuel beds, dryers for granules, etc.

The experiments under steady state were conducted by blowing air through a layer of polished steel balls supported within a glass tube and heated by induction so as to generate a uniform amount of heat within all points of the charge.

The instrumentation included a psychrometer; venturi flowmeter; fan; micromanometers to read the pressure drop across the charge; various thermocouples inserted in the charge balls and in the airstream; and electrical measuring apparatus calibrated to read the energy input at the heating inductor.

Results, charts, and empirical relationships covering most variables are given and explained with a critical comparison to a twenty-item bibliography including Russian and some prewar American works. B. Posniak, USA

3102. **Freeston, H. G., Roberts, J. D., and Thomas, A., Crankcase explosions: An investigation into some factors governing the selection of protective devices,** Instn. mech. Engrs. Preprint, 14 pp. + 2 plates.

Remedies which have been suggested for crankcase explosions include the use of warning devices to detect overheating in the crankcase, and the provision of relief vents to limit the explosion pressure to a safe value.

Data were obtained experimentally at the Thornton Research Centre for the conditions necessary for the ignition of mists of lubricating oil in air in order that the margin of safety offered by a detector of overheating might be more fully assessed.

The venting of explosions was also studied both on a rig scale and in the crankcase of a diesel engine of 1100 hp. The results indicated that it would not be practical to provide sufficient venting area on an engine of this size to limit the maximum explosion pressures to a safe value for the more violent explosions which might occur. For venting to provide complete protection, it would seem necessary to take measures to prevent the flame from spreading throughout the whole crankcase.

From authors' summary

3103. **King, R. O., and Rand, M., The oxidation, decomposition, ignition, and detonation of fuel vapors and gases. XXVII. The hydrogen engine,** *Canad. J. Technol.* **33,** 445–469, 1955.

Previous attempts to run an Otto-cycle engine on hydrogen have failed because it was considered this fuel "detonates easily." The experiments described do not support this view. When arrangements were made to prevent preignition, exceptionally good performance data were obtained. By keeping all surfaces as cold as possible and using a lubricating oil which does not interfere with the combustion one can cover a wide range of mixture strength, when the spark setting is adjusted accordingly. This is illustrated by oscilloscope diagrams and several graphs give the actual performance figures. From authors' summary

Acoustics

(See also Revs. 2918, 2948, 3049)

3104. **Bancroft, D., Measurement of velocity of sound in gases,** *Amer. J. Phys.* **24,** 5, 355–358, May 1956.

A method of determining the velocity of sound by observing the frequencies of radial oscillation of a gas confined in a spherical cavity is described. The accuracy appears to be comparable with that of the best methods available to date. A few preliminary results are quoted for argon, oxygen, and CO_2. The possibility of utilizing the measurements for determination of the absolute temperature of the ice point is discussed.

From author's summary by A. H. Hausrath, USA

3105. **Helberg, H.-W., Sound field in rectangular tubes** (in German), *Akust. Beihefte* no. 2, 578–586, 1954.

3106. **Pritchard, R. L., Maximum directivity index of a linear point array,** *J. acoust. Soc. Amer.* **26,** 6, 1034–1039, Nov. 1954.

The gain of a linear array of point sources is maximized by varying the individual source strengths subject to the constraint that the broadside field is constant. The method of the Lagrangian multiplier is used to obtain a determinate set of algebraic equations for the strengths for a given number and spacing of elements. In particular, for integral-half-wave length spacing, the equations yield Rayleigh's result: uniform strengths are required for a maximum gain equaling the number of elements N. Graphs of the maximum gain (G) as a function of R = spacing ratio line wave length (from 0 to 1) are given for $N = 3$, 5, and 7 and compared with the gain resulting from uniform strengths (U). For $1/2 < R < 1$, $G = U$; for $R < 1/2$, G may be much greater. The radiation patterns corresponding to the above G's and U's are approximately equal for $1/2 < R < 1$. On the other hand, for $R < 1/2$, the amplitudes of the minor lobes are greater, and the half-width of the major lobes are less for the G curves; this "super-gain" requires large out-of-phase excitation of neighbors, (and usually results in practical difficulties involving decreased efficiency, stringent tolerances, and narrow band widths).

This paper should prove useful in the practical design of directive arrays. V. Twersky, USA

3107. **Krishnamurty, K., Acoustic radiation from two-dimensional rectangular cutouts in aerodynamic surfaces,** *NACA TN* 3487, 33 pp., Aug. 1955.

Subsonic and supersonic flow of air past rectangular cavities cut into a flat surface were studied. The cavities were found to emit a strong acoustic radiation. These acoustic fields were investigated by means of schlieren observation, interferometry, and hot-wire anemometer.

The frequency of the sound, as measured by a hot-wire, at a fixed depth of the gap was found to be inversely proportional to the breadth. No simple relationship was noticed between the Mach number M_∞ and a nondimensional frequency S based on the gap breadth and the free-stream velocity.

The intensity of the radiation was conveniently measured by means of an optical interferometer of the Mach-Zehnder type. At a Mach number of 0.8, a unit fringe shift was found to correspond to an intensity level of 160 decibels. Fringe shifts equal to and greater than 1 were observed for the radiations from these cavities, indicating that the acoustic fields were very intense.

From author's summary

3108. **Kornhauser, E. T., and Raney, W. P., Attenuation in shallow-water propagation due to an absorbing bottom,** *J. acoust. Soc. Amer.* **27,** 4, 689–692, July 1955.

The attenuation of normal modes in a two-layered fluid medium due to absorption in the lower semi-infinite layer is calculated under the assumption that this absorption is small. The result obtained gives the attenuation coefficient for each mode as the product of the absorption coefficient for plane waves in the absorbing layer and a dimensionless function of wave length and layer depth. Curves of this dimensionless function are plotted for the first five modes in cases typical of propagation in shallow water over a sand and a mud bottom. The results indicate a very rapid decrease in attenuation as the frequency rises above the cutoff frequency, where it is just equal to the bottom absorption. At higher frequencies, the attenuation varies as $(n^2\lambda^3/H^3)\alpha$, where n is the mode number, λ the wave length, H the water depth, and α the absorption coefficient in the bottom (α may itself be a function of frequency). The latter result is also derived by a less rigorous method.

From authors' summary

3109. **Loeber, A. P., and Hiedemann, E. A., Investigation of stationary ultrasonic waves by light refraction,** *J. acoust. Soc. Amer.* **28,** 1, 27–35, Jan. 1956.

3110. **Kaspar'iants, A. A., Propagation of sound waves in a viscous gas in the presence of heat conductivity** (in Russian), *Prikl. Mat. Mekh.* **18,** 6, 729–734, 1954.

To solve the equations of sound propagation in a viscous gas a stationary field is assumed, time entering as factor $e^{i\sigma t}$. In the final system there is an equation containing only one variable $S = (\rho - \rho_0)/\rho_0$; other variables can be expressed in terms of S. In further analysis, small values of second order are neglected, and general results are applied to the case of attenuation of a stationary sound field produced by a flat radiating piston harmonically oscillating in an infinite mass of viscous gas. The use of stationary phase method leads to a solution showing the similarity of this field pattern to that of a nonviscous gas. The damping coefficient for sound waves is given. P. Bielkowicz, USA

3111. Strasberg, M., Gas bubbles as sources of sound in liquids, *J. acoust. Soc. Amer.* **28,** 1, 20–26, Jan. 1956.

Author shows that sound generated by small gas bubbles entrained in a liquid is mainly associated with volume pulsations of the bubbles; the sound pressure generated by shape pulsations being negligible. Theory used is a linear approximation and is applied for describing sound pressures and spectra generated in bubble formation at a nozzle, bubble coalescence or splitting, and when bubbles are entrained past bodies and constructions. Limits of the applied theory and the effects of nonlinearities are indicated.

Reviewer believes this article is an important contribution in understanding the phenomenon of sound generated by gas bubbles entrained in a liquid. J. H. van de Stadt, Holland

3112. Withington, H. W., Silencing the jet aircraft, *Aero. Engng. Rev.* **15,** 4, 56–63, 84, Apr. 1956.

This is, in effect, a progress report of the jet-silencing program currently in progress. After three months of actual testing with seven full-scale nozzles and over 40 small-scale nozzles, the noise from the jets can be reduced by amounts which could result in jet aircraft considerably quieter than present-day operating transports. It is not possible at this time to predict the inevitable trade between performance penalty and noise reduction. Further refinements will be required before the optimum combination of airplane and engine performance characteristics with the desired amount of noise reduction has been achieved. (*Extracted from author's conclusions*).

Paper describes the high-frequency measuring system used. Sound-level and directivity plots are given for a number of scale-model and full-size nozzles. Sound-level plots of jet aircraft noise during a high-speed taxi test are given; comparison with noise from propeller-driven aircraft is made. Predicted sound levels on the ground during takeoff are appreciably less for the jet-silenced plane than for a comparable propeller-driven plane.

J. N. Brennan, USA

3113. Vance, D. H., The use of vibration and shock control in reducing noise levels, *Noise Control* **2,** 2, 64–72, Mar. 1956.

3114. Hudson, G. E., Thrust on a piston driven half-open tube, *J. acoust. Soc. Amer.* **27,** 3, 406–415, May 1955.

The motion of the gas in and near a tube open at one end and closed at the other by a piston driven in nearly sinusoidal motion is investigated. Experiments disclose that, when the frequency of the piston motion is near the fundamental acoustic resonance frequency of the gas column, an intermittent air jet associated with a system of vortex rings is expelled from the tube's open end. This results in an appreciable average thrust on the tube which, however, is much less than might be expected from purely acoustic considerations. A theoretical analysis has been made which takes account of the intermittent vortex and jet formation, as well as of the acoustic radiation field. The analysis leads to formulas for the average thrust and power dissipation and to a relation between the driving frequencyand the amplitude of the gas motion at the open end. An empirical parameter κ, which represents the fraction of energy dissipated in vortex motion, appears in these formulas. It is found possible to evaluate this parameter in terms of the observed thrust at resonance. From this datum the curve of thrust versus driving frequency is calculated and is found to agree well with experiment. From author's summary

Ballistics, Detonics (Explosions)

(See also Revs. 2826, 3038)

3115. Reece, J. W., Joseph, R. D., and Shaffer, D., Ballistic missile performance, *Jet Propulsion.* **26,** 4, 251–255, Apr. 1956.

Choice of good design of missiles depends upon judicious compromise among many design factors, and is complicated by the tedious calculations of trajectory necessary to evaluate relative importance of different aspects of design. Authors present a simplified method for evaluating maximum range of intermediate range (50 to 500 nautical miles), one-stage ballistic rockets, which is particularly adaptable to problems where selection of design parameters is more important than determination of the exact range. Paper includes graphs for use in drag corrections on the vacuum range. E. W. Price, USA

3116. Vertregt, M., A method for calculating the mass ratios of step-rockets, *J. Brit. interplanetary Soc.* **15,** 2, 95–97, Mar.-Apr. 1956.

3117. Terwilliger, D. D., and Granros, G. J., Sun-follower for high altitude sounding rocket, *Jet Propulsion* **26,** 4, 275–277, Apr. 1956.

3118. Kavanau, L. L., Base pressure studies in rarefied supersonic flows, *J. aero. Sci.* **23,** 3, 193–207, 230, Mar. 1956.

Paper presents essentially only data available to date on base pressure in rarefied gas flows. Measurements were made on cone-cylinder body at Reynolds numbers between about 200 and 7000. Mach numbers were 2 and 4. Results are interesting in that they show in many cases opposite trends to those observed at higher Reynolds numbers where slip-flow does not exist. For example, present measurements in slip regime show the base pressure to increase substantially with increasing Reynolds number and to be markedly nonuniform over the entire base area. To reviewer's knowledge, these two phenomena have not been observed in the continuum regime. The interference effects of support length and support diameter were investigated in detail. Limited pressure distributions on model and in wake were made. Theoretical calculation of base pressure in free molecule flow is presented.

D. R. Chapman, USA

3119. Granstrom, S. A., Loading characteristics of air blasts from detonating charges, *Trans. roy. Inst. Technol. Stockholm* no. 100, 93 pp., 1956.

Paper is author's thesis for degree of Licentiate of Technology, and aims at evaluating structural loadings resulting from TNT air detonations. Author reviews shock hydrodynamics and geometric scaling laws for high explosive charges. Overpressures, total impulses, and wave lengths are computed and graphed as functions of charge weight and distance. These results are based on approximate calculations which start from experimentally observed shock-wave velocities [*Tidskrift för Kuslartilleriet* **5,** 44–62, 1947], and which are subject to the following assumptions: (1) Shock waves are perfectly spherical—ground effects are neglected. (2) Air and the gaseous products of explosion do not mix. (3) Once the shock velocity is given, the only important requirements are that mass be conserved and equations of state satisfied.

The equation of state for the gaseous products of TNT explosions is taken from *Proc. roy. Soc. (A)* **194,** 480–507, 1948. No mention is made of recent numerical solutions of spherical blast waves, carried out in the USA by Brode and others—i.e., *J. appl. Phys.* **26,** 766–775, June 1955. J. H. Huth, USA

Soil Mechanics, Seepage

(See also Revs. 2841, 2907, 2910, 2911, 2962, 3130, 3131, 3132)

3120. Runyan, W. R., and Anderson, R. E., Mechanical impedance measurements of soils, *J. acoust. Soc. Amer.* **28,** 1, 73–76, Jan. 1956.

Values of driving-point impedances on ground surfaces are of interest in problems involving propagation of disturbances through soil, exploration of underlying strata, and dynamic reactions of foundations. This concise paper describes a method of measuring the mechanical impedance of soil by vibrating a piston and using piezoelectric devices to determine forces and velocities. Measured values are plotted from 50 to 350 cps and show within limits effects of water content, soil character, and source diameter. When compared with some previous theoretical work, theoretical values were found to be 2 to 3 times the measured values — not bad agreement considering the many approximations made.

G. J. Tauxe, USA

3121. Hobbs, N. B., Some notes on the determination of stresses beneath loaded areas, *Trans. S. African Instn. civ. Engrs.* 253–256, Aug. 1955.

In the calculation of stresses in soil masses beneath loaded areas it is seldom possible to directly apply rigorous methods of theory of elasticity, and it is therefore necessary to make simplifying assumptions. Authors show in some examples of practical importance how simplifications of mathematical nature can be made and justified by comparison with known rigorous solutions.

E. Rathgeb, Argentina

3122. Boreli, M. M., Contribution to the study of porous media (in French), *Publ. sci. tech. Min. Air, France* no. 305, 129 pp., 1955.

The four subjects studied are (1) infiltration through dikes, (2) drain placed on impermeable layer, (3) cores in dikes, (4) well which does not reach impermeable stratum.

Following chapter on plane flow and conformal transformations is a detailed study of singular points. For (2), graph is developed relating length of drain and flow. Existence of a critical pressure beyond which it is impossible to increase further the discharge without creating irregularities in flow is demonstrated. Hydroelectric analogy, applied to (3), was used to study (a) flow through dike with impermeable core and upstream drain and (b) flow through dikes with impermeable cores of different heights.

A variant of relaxation method, using residues of irregular stars, is given and applied to (4).

V. L. Dutton, Canada

3123. Al'perin, L. B., Passive pressure of loose granular material on retaining walls (in Russian), *Inzhener. Sbornik, Akad. Nauk SSSR* **21,** 156–162, 1955.

The cohesionless granular material supported by a retaining wall and having a constant free slope is assumed to be everywhere on the verge of sliding. The plane state of stress in this material is described by three stress components assumed proportional to the radius drawn from the top corner of the wall. Constant stress obliquity is assumed on the plane of the wall. Using two new variables, satisfying identically the equation of maximum obliquity, the problem is solved by numerical integration, after overcoming some difficulties in the boundary conditions at the wall and at the border line of the two stressed zones, into which the backfill region is divided.

Calculation performed for a vertical retaining wall shows that the elementary Coulomb theory overestimates the passive pressure. For a large value of the angle of internal friction this error of Coulomb's theory becomes very great.

A. Hrennikoff, Canada

3124. McNulty, J. F., Thrust loading on piles, *Proc. Amer. Soc. civ. Engrs.* **82,** SM2 (*J. Soil Mech. Found. Div.*), Pap. 940, 26 pp., Apr. 1956.

Lateral load tests were performed on two separate projects for the National Advisory Committee for Aeronautics, Langley Field, Virginia. The first project utilized concrete cast-in-place piles, heads not fixed, embedded in a medium dense silty-sand. The second project employed fixed-end timber piles embedded in a medium sandy clay. The field data and an approximate method of analysis are presented and discussed. Graphs of a preliminary nature indicating the relationship between deflection and load for various soil conditions are included.

From author's summary

Micromeritics

(See also Revs. 2910, 2911)

3125. Dannenberg, R. E., Gambucci, B. J., and Weiberg, J. A., Perforated sheets as a porous material for distributed suction and injection, *NACA TN* 3669, 27 pp., Apr. 1956.

The use of distributed suction as a means for controlling the behavior of the boundary layer, and of injection for alleviating aerodynamic heating, has recently received a great deal of attention. The materials used generally depend on the specific application and are either sintered metals, layers of wire cloth, or perforated metal sheets. This report describes the results of an extensive set of experimental measurements of the resistance to air flow of a series of perforated metal sheets having open areas varying from less than 1% up to about 41%. These results are given in graphical form. It was found that the permeability of a perforated sheet is governed principally by the open-area ratio. The number of holes per unit area, sheet thickness, and the shape of the individual holes had little or no effect on the permeability.

J. Persh, USA

3126. Gusein-Zade, M. A., Stationary flow in two strata divided by a slightly permeable wall (in Russian), *Izv. Akad. Nauk SSSR Otd. tekh. Nauk* no. 12, 134–136, Dec. 1954.

Classical treatment of flow problems assumes impervious boundaries. In oil extraction, the problem is met of flow with leakage through slightly pervious boundaries. Author illustrates approach in a two-dimensional case.

Problem is reduced to that of finding two functions p_i (x, y) for the pressure, harmonic over rectangular regions ($i = 1, 2$). On two opposite sides p_i takes arbitrary constant values. At contact with wall, normal derivative of p_i is linearly related to difference of pressure across wall. A sine transform is applied to differential equation and boundary conditions. Expressions obtained for p_i are series with terms of the form $\mathrm{chn}(b+y)\cdot \sin nx$.

Author states that method holds for nonstationary and for axisymmetrical flows, with similar boundary conditions. Solutions are indicated in Schelkachev and Gusein-Zade: *Neft. Khoz.* **12,** 1953.

G. H. Beguin, Switzerland

3127. Arkhangel'skiĭ, V. A., Computation of one-dimensional motion of underground water by means of finite differences (in Russian), *Inzhener. Sbornik, Akad. Nauk SSSR* **16,** 203–210, 1953.

3128. Aris, R., On the dispersion of a solute in a fluid flowing through a tube, *Proc. roy. Soc. Lond. (A)* **235,** 1200, 67–77, Apr. 1956.

Sir Geoffrey Taylor has recently discussed the dispersion of a solute under the simultaneous action of molecular diffusion and variation of the velocity of the solvent. A new basis for his analysis is presented here which removes the restrictions imposed on some of the parameters at the expense of describing the distribution of solute in terms of its moments in the direction of flow. It is shown that the rate of growth of the variance is proportional to the sum of the molecular diffusion coefficient D, and the Taylor diffusion coefficient $\kappa a^2 U^2/D$ where U is the mean velocity and a

is a dimension characteristic of the cross section of the tube. An expression for κ is given in the most general case, and it is shown that a finite distribution of solute tends to become normally distributed. From author's summary by S. Corrsin, USA

3129. Huard de la Marre, P., Exact expressions for infiltrations in three-dimensional dams with vertical wall (in French), *C. R. Acad. Sci. Paris* **242**, 9, 1125–1127, Feb. 1956.

The formulas of Dupuit for the filtration in an aquifer on an impervious horizontal body are strictly established by M. Perès, who applies the formula of Green with two harmonic functions ϕ, potential of the filtration flow, and x for a flow limited by two vertical parallel plans, or $\log (x^2 + y^2)$ for two concentric cylinders.

Author presents extensions of this strict solution.

L. J. Tison, Belgium

3130. Guzov, M. Z., Computation of seepage through earth dam with an apron and a cutoff (in Russian), *Gidrotekh. Stroit.* **22**, 10, 31–32, 1953.

Apron is an inclined core resting on upstream face; it is continued by a vertical cutoff extending up to the impervious base. Author wishes to simplify formula [Uginčus, Moscow, 1940] involving cubic equation for intercept of seepage line with apron in case of full reservoir. Body of dam is homogeneous; apron is as thick and as permeable as cutoff. He equates amount of seepage across apron plus cutoff to that through body of dam. He gets formula for intercept of extended seepage line with vertical through cutoff. Equation for seepage line is of the form $y = (a + bx)^{1/2}$.

Reviewer finds treatment simple and much idealized as compared to cases met in practice.

G. H. Beguin, Switzerland

3131. Mkhitaryan, A. M., Seepage through an earth dam with pervious foundation and sloped water retaining wall (in Russian), *Inzhener. Sbornik, Akad. Nauk SSSR* **16**, 211–212, 1953.

3132. Meyer, H. I., and Searcy, D. F., Analog study of water coning, *J. Petr. Technol.* **8**, 4, 61–64, Apr. 1956.

Geophysics, Meteorology, Oceanography

(See also Revs. 2907, 2973, 3078)

Book—3133. Schneider-Carius, K., Meteorology [Wetterkunde Wetterforschung], Munchen, Verlag Karl Alber, 1955, xvi + 423 pp. DM 27.50.

This interesting book is a history of development of meteorology from first steps of human culture to modern science. Contents: Old weather knowledge. Beginning of observations. Physical theories and laws. Presentation of observations. Origin of dynamic meteorology. Thermodynamics of atmosphere. Physics of clouds and precipitation. Color of sky. Electric field. Rhythms in variations. Climatology. Energetics of atmosphere. Free atmosphere and boundary layer. Synoptics. Weather forecasting. Realm of contemporary meteorology is presented in attractive form of its historical development, based on original sources and documents.

Although book is intended for a wide circle of readers, it is written on high scientific level and is extremely rich in facts, citations, and discussions. Excellent appendixes include sources, systematic bibliography (31 p.), biographical dictionary (16 p.), author and subject indexes. Publisher is to be praised for splendid accomplishment. Book is of great value as a good introduction to meteorology. S. Kolupaila, USA

3134. Fleagle, R. G., The temperature distribution near a cold surface, *J. Meteor.* **13**, 2, 160–165. Apr. 1956.

Author applies a method previously suggested by him [*Bull. Amer. meteor.* **31**, 51–55, Feb. 1950] to measure the temperature gradient near a cold water surface. Method depends on bending of light rays in a nonuniform temperature field. Sensitivity of method is such as to reveal small, previously undetected temperature anomaly near the cold surface. This anomaly is probably unimportant from an engineering standpoint, but method holds interest because of its implications in surveying and as a tool for measuring temperatures over either hot or cold surfaces not accessible to ordinary thermometers. F. I. Badgley, USA

3135. White, R. M., and Cooley, D. S., Kinetic-energy spectrum of meridional motion in the mid-troposphere, *J. Meteor.* **13**, 1, 67–69, Feb. 1956.

Kinetic-energy spectra are derived for the meridional motion at latitude 45°N from 500 mb maps for three winter months. Authors draw attention to a tendency for two maxima of energy around wave numbers four and eight. Identification of these maxima with specific types of atmospheric disturbance is inconclusive.

J. S. Sawyer, England

3136. Mange, P., Diffusion processes in the thermosphere, *Ann. Geophys.* **11**, 2, 153–168, 1955.

Previous contributions to diffusion theory for minor constituent gases in the upper atmosphere deal largely with isothermal conditions. Author considers the effect on the distribution of minor constituents of an increasing temperature with height above the 100-km elevation. It is shown that the diffusion equilibrium distribution can be significantly altered by the effect of thermally induced diffusion for minor constituents of low molecular weight, such as hydrogen or helium. The transient state diffusion of different constituents in the absence of thermal diffusion is studied. The effect of a source and sink with and without thermally induced diffusion is studied. The diffusion of molecular oxygen from an original state of photochemical equilibrium is discussed and it is shown that the upward diffusive transport of oxygen tends toward a diffusion equilibrium distribution in spite of the dissociation process. Mixing due to turbulence, if effective above 100 km, would tend to reinforce the upward transport by diffusion. Effect of turbulent transport of constituents is not discussed, except in respect to what the author refers to as the mixing equilibrium state, which is the distribution of the constituent which would presumably exist if the mixing were complete. This state is regarded as one in which the proportion of the minor constituent to the major constituent is the same at all levels, but the absolute value of each constituent decreases with elevation.

R. G. Reid, USA

3137. Lake, H., A meteorological analysis of clear air turbulence (A report on the U. S. synoptic high-altitude gust program), U. S. Air Force, Cambridge Research Center, Geophys. Res. Pap. 47, 63 pp., Feb. 1956.

The U. S. Synoptic High-Altitude Gust Program lasted from 1200Z 18 March 1953 to 1200Z 21 March 1953. Observations were made within plus or minus one hour of 1600Z and 0400Z, and at the altitudes of 30,000 and 39,000 ft, which approximate the mean heights of the 300- and 200-mb surfaces, respectively. A total of 598 reports of occurrence or nonoccurrence of turbulence was received. A total of 180 upper air maps were drawn for the three pressure surfaces 300, 250, and 200mb and at the two reporting times, 0300Z and 1500Z. Upon careful inspection, no obvious relation between turbulence and the dominant meteorological features of the charts was apparent, contrary to the current idea on the meteorological aspect of clear air turbulence. The clear air turbulence experienced by pilots is concluded to be a result of the influences of pertinent meso-scale atmospheric flows, the plane's characteristics, such as its speed and attitude, as well as the sensory reaction of the pilot. Statistical results indicate that the

vertical gradient of the horizontal kinetic energy is related to clear air turbulence at the 95% confidence level, and turbulence would be more likely associated with negative shear (wind speed decreasing with height).

Reviewer believes that author should make some comments on reviewer's earlier work [*Geofisica pura e applicata* Milano, **20**, 56–61, 1951] in which reviewer tried to show how turbulence is, in general, associated with negative shear.

H. Arakawa, Japan

Lubrication; Bearings; Wear

(See also Rev. 2945)

3138. Smith, M. I., and Fuller, D. D., Journal-bearing operation at superlaminar speeds, *Trans. ASME* **78**, 3, 469–474, Apr. 1956.

An apparatus for measuring friction in a journal bearing as a function of Reynolds number is described. Water was used as the test lubricant because of its low viscosity, fairly good viscosity index, and high specific heat. Average temperature in the film was arbitrarily assumed to be the arithmetic mean of the inlet and outlet temperatures. The latter was calculated from power loss in the bearing, lubricant flow rate, and thermal gradients in the bronze.

Confirmation was obtained of calculated data for finite bearings in the laminar region. Pressure profile in bearing at eccentricity ratio of 0.8 agrees reasonably with Gümbel's calculation for an infinite bearing. On the other hand, up to an eccentricity ratio of 0.4, friction torque at 1800 rpm follows Cameron and Wood's curve. Above the critical speed, load-carrying capacity, pressure profiles, and the friction torque were measured. Authors remark that "the turbulent bearing exhibited an astonishing variation of friction with load; that is, with increasing load, it became less difficult—at least up to an optimum point—to drive the journal."

It may well turn out that this is one of the first experimental demonstrations of the "shear orientation" of water molecules. Authors' own conclusion is given in more conventional terms: "For loaded bearings, laminar pressure gradients in the load-carrying film were maintained to speeds some five times greater than the critical speed."

R. Schnurmann, England

3139. Johnston, R. C. R., and Kettleborough, C. F., An experimental investigation into stepped thrust-bearings, Inst. mech. Engrs., Preprint 10 pp., 1956.

Lord Rayleigh considered the effect of a sudden change in the thickness of a lubricant film and, neglecting side leakage, concluded that the load-carrying capacity would be considerable.

Archibald has analyzed the finite width case and suggested that a semi-circular step concave to the direction of motion would be preferable to a straight step. Kettleborough, using Kingsbury's electrical analogy, has solved the case of the sector-shaped pad having various forms of step including the semicircular form and a modified form which he claimed gave highest performance.

Experimental confirmation of this result is provided by the present authors. Three pads were hydraulically loaded in a vertical spindle testing machine. The variation in oil-film thickness was measured using an air gaging system. The pads were made of steel and the runners of 10% leaded bronze. This inversion of the conventional arrangement whereby the member under constant load is made of softer more conformable material while the continuous member is made of the harder material was justified by these workers on the grounds that wear of the pads should be minimized in order to preserve the stepped form and thus the efficiency of the bearing.

The steps, approximately 0.001 in. high, were formed by electrodeposition of chromium, a mask being used to cover that part of the area of the pad which formed the entry region leading up to the step. Surfaces were carefully spot ground, using a wobble wheel where possible so that no point was ground continuously. A high degree of accuracy was obtained, the raised portion of the step being flat within ± 0.00005 inch.

Over the range of variables tested, minimum film thickness varied from 0.0005 and 0.0025 in., being about 0.0005 in. thicker than the corresponding minimum film thickness of a tilting pad bearing.

The coefficients of friction of all but one of the stepped pad forms tested were higher than those obtained for the Michell bearing.

F. T. Barwell, Scotland

3140. Charron, F., Study of lubrication under conditions of rapidly variable pressure (in French), *Publ. sci. tech. Min. Air, France* no. 310, 27 pp., 1956.

Microscopical examination by the author of thin layers of plastic lubricants has shown that they are formed, in general, of optical anisotropic particles dispersed in a viscous liquid medium. If a movement is established inside a film of such lubricants, some orientation of the particles occurs. Author explains, according to conclusions he published previously, that, for such materials, two different friction coefficients have to be distinguished, one internal between adjacent layers, and one between the wall and the adjacent layer, the static value f_1 of the former being smaller than the static value f_2 of the latter.

Different types of lubricants, including plastic lubricants, have been submitted to two series of operations: (1) Flow in capillary tubes under the action of a large pressure suddenly applied; (2) flow between the two normal faces of a cylinder turning between two fixed cylinders charged by a pressure suddenly applied. The recording diagrams showing the behavior of the different lubricants are interpreted and discussed. The instruments used are especially well fitted for classifying oils and fats according their ability to resist to high pressures.

D. De Meulemeester, Belgium

3141. Özdas, N., and Ford, H., Oil transfer and cooling in ring-oiled bearings, Part I, Part II, *Engineering* **180**, 4674, 268–271, Aug. 1955; **180**, 4682, 570–573, Oct. 1955.

Data on effect of ring immersion, shaft speed, journal diameter, and oil viscosity on ring speed and oil delivery are presented. A nondimensional plot involving delivery, ring and shaft diameters, shaft speed, and oil viscosity gives a useful correlation for determining oil delivery.

Friction torque, bearing temperature rise, and bearing surface temperature data are presented as functions of load and speed. Bearing heat dissipation calculations are presented.

W. J. Anderson, USA

3142. Aoki, S., Utsumi, T., and Nemoto, S., Measurement of ball bearing clearance, *J. mech. Lab., Tokyo* **1**, 2, 57–58, 1955.

3143. Hasselgruber, H., Calculation of elastic and damped bearing systems of high revolution shafts, *Motortech. Z.* **15**, 12, 373–376, Dec. 1954.

Marine Engineering Problems

(See also Revs. 2839, 2857)

3144. Korvin-Kroukovsky, B. V., Stern propeller interaction with a streamline body of revolution, *Inter. Shipbldg. Progr.* **3**, 17, 3–24, Jan. 1956.

This is the second paper [for first part see title source **1**, 4, 170–178, 1954] of a series dealing with numerical calculation of wake fraction and thrust deduction of a ship propeller. Fluid velocities of the inflow of a free-running propeller, represented by

one and two single cylindrical vortex sheet, respectively, are calculated by vortex theory assuming infinite theory number of blades and neglecting propeller race rotation. Results are found to be identical with those obtained by assuming uniform distribution of sinks over propeller disk area. Assuming one sink at the propeller disk, like Dickmann, Martinek and Yeh, is justified for large distances (over five radii) only. Propeller-hull interaction is treated for the axisymmetric case without free surface. The stern portion of the hull of the U.S. Airship Akron is approximated by six sinks and six sources. Wake fraction and thrust deduction with and without boundary-layer effects, using experimental work referred to, are evaluated by theoretical methods with certain assumptions. Their numerous components are widely discussed, with some conclusions for further work on this subject.

H. Thieme, Germany

3145. Christensen, H., Experimental determination of propeller torque variations on ship models, *Inter. Shipbldg. Progr.* **3,** 20, 229–239, Apr. 1956.

3146. Christopher, K. W., Effect of shallow water on the hydrodynamic characteristics of a flat-bottom planing surface, *NACA TN* 3642, 36 pp., Apr. 1956.

The effects on the planing characteristics of the clearance between a flat-bottom planing surface and the tank bottom are presented. The range of trims investigated was from 4° to 20° for wetted-length—beam ratios of 0.4 to 6.4. Each condition was investigated over a range of clearances of from 0.2 to 1.6 beams. All the measured values increased with decreasing clearance. A description of the monorail and its associated apparatus is included. From author's summary

3147. Lap, A. J. W., Diagrams for determining the resistance of single-screw ships, *Inter. Shipbldg. Progr.* **1,** 4, 179–193, 1954.

With the aid of the results of many model experiments which have been carried out by the Netherlands Ship Model Basin for upwards of twenty years, attempts have been made at devising a method to determine ship resistance. Investigations have been made with a view to finding out what parameters are of importance for characterizing the ship form and, next, to ascertaining what influence each of these parameters has on ship resistance. Diagrams are provided for determining the residuary resistance as a function of speed. The construction of these diagrams is described. Moreover, some conclusions are mentioned which could be drawn during the investigations. Finally, the total efficiency of propulsion is given as a function of the number of revolutions of the propeller and the ship's length. By these means it is possible to derive the ship's power under service conditions from the resistance of the smooth ship, after various allowances have been taken into account. From author's summary

3148. Korvin-Kroukovsky, B. V., On the numerical calculation of wake fraction and thrust deduction in a propeller and a hull interaction, *Inter. Shipbldg. Progr.* **1,** 4, 170–178, 1954.

A ship-propeller interaction theory is presented in the form of computational procedure for evaluation of the nominal and effective wake fractions and of propeller thrust deduction, taking into account the contributions of potential flow, viscous boundary layer, and ship wave-making. This method will give the *distribution* of the *effective* wake, and thereby will permit a more precise design of wake-adapted propellers, the action of which heretofore had to be inferred from the experimental data on the distribution of *nominal* wake.

Calculations are based on representing the propeller race by an array of cylindrical vortex sheets, thus accounting correctly for mean axial and radial flows, but neglecting the time fluctuations and rotation of the propeller race. Any form of the radial thrust distribution can be specified. The hull is represented by distribution of sources and sinks, in the evaluation of which the velocity field of the propeller and the ship wave flow are taken into account. The ratios of computed effective and nominal wakes can be used then as correction factors to observed nominal wakes. For submerged bodies of revolution, calculations can be made using desk machines. For a surface ship, the use of punched card computing machines is visualized. From author's summary

3149. Silovic, S., and Fancev, M., Resistance and propulsion. Measurements on the motorship "Rijeka", with their attempted practical application, *Shipbuilder* **63,** 576, 264–267, Apr. 1956.

3150. Van Lammeren, W. P. A., Van Manen, J. D., and Lap, A. J. W., Scale effect experiments on Victory ships and models. Part. 1. Analysis of the resistance- and thrust-measurements on a model family and on the model boat D. C. Endert, Jr., *Inter. Shipbldg. Progr.* **3,** 18, 77–108, Feb. 1956.

This paper deals with the resistance and thrust measurements on a series of nine geometrically similar Victory ship models, ranging in scale from 160 to 18, and on a Victory model boat of scale 6. The first part of the paper gives a description of the ship, the propeller, the models, the test apparatus, the test performance, the instrument calibration and the necessary corrections.

The measured resistance and thrust values of the model boat are correlated with the results of the tests with the models, using three methods, i.e., the Schoenherr, the Hughes, and the Lap-Troost extrapolators. The results of experiments for the determination of the wall interference are given. These experiments are on the principle of the "image method." An attempt is made to analyze the thrust-deduction factors as a function of the Reynolds number.

Conclusions and a program for the continuation of the tests are presented at the end of the paper.

From authors' summary

3151. Hadler, J. B., and Hinterthan, W. B., Wax model construction at the David W. Taylor Mod. Basin, *David W. Taylor Mod. Basin Rep.* 930, 19 pp., June 1955.

This report gives an account of the development of a satisfactory wax blend for the manufacture of ship models at the David Taylor Model Basin. The blend selected consists of 30% refined paraffin (Aristowax), 32.5% hydrogenated castor oil (Opalwax), and 37.5% n-butyl methacrylate (Lucite 44). This blend is suitable for use in the Washington, D.C. climate for the manufacture of models up to 33 feet in length. Its use results in model costs which are about 55% of those for similar wood models, and in a reduction in manufacturing time of almost two weeks. The manufacturing methods in use with this wax blend are also outlined in some detail in this report. From authors' summary

Books Received for Review

DEN HARTOG, J. P., Mechanical vibrations, 4th ed., New York, McGraw-Hill Book Company, Inc., 1956, xi + 436 pp. $9.

DRYDEN, H. L., MURNAGHAN, F. P., and BATEMAN, H., Hydrodynamics, New York, Dover Publications, Inc., 1956, 634 pp. $2.50 (paperbound).

EINSTEIN, A., Investigations on the theory of the Brownian Movement, New York, Dover Publications, Inc., 1956, 119 pp. $1.25.

ERDÉLYI, A., Asymptotic expansions, New York, Dover Publications, Inc., 1956, 108 pp. $1.35 (paperbound).

GARDEL, A., Chambres d'équilibre, Lausanne, France, F. Rouge & Cie S. A., Librairie De L'Université, 1956, 162 pp. Fr. 24.85 (paperbound).

HERTZ, H., The principles of mechanics presented in a new form, New York, Dover Publications, Inc., 271 pp. $1.75 (paperbound).

KNOPP, K., Infinite sequences and series, New York, Dover Publications, Inc., 1956, 186 pp. $1.75 (paperbound).

SEELYE, E. E., Foundations—Design and practice, New York, John Wiley & Sons, Inc., 1956, xvi + 425 pp. $16.

SNEDDON, I. N., Special functions of mathematical physics and chemistry, New York, Interscience Publishers, Inc; London and Edinburgh, Oliver and Boyd, 1956, viii + 164 pp. $1.75.

D. C. Drucker and Joseph Kestin Named Technical Editors of Journal of Applied Mechanics

On Oct. 1, 1956, technical editorship of the *Journal of Applied Mechanics* will be transferred from J. M. Lessells to D. C. Drucker and Joseph Kestin of Brown University. Dr. Drucker and Dr. Kestin, as technical editors of the Journal will take over gradually the present duties of the Editorial Board of the Applied Mechanics Division of The American Society of Mechanical Engineers.

Professor Lessells will continue as Honorary Editor, assisting the technical editors in a consulting capacity. Since the establishment of the Journal, Professor Lessells has been technical editor. He brought to the magazine the benefits of many years of experiences as a practicing engineer both in England and the United States. His interest in this professional journal in the applied mechanics field ran parallel with his teaching career at the Massachussetts Institute of Technology. Now he is president of Lessells and Associates, Inc. of Boston, Mass. He is himself the author of "Strength and Resistance of Metals" and coauthor of "Applied Elasticity." In 1938 he was editor of the "Stephen Timoshenko 60th Anniversary Volume." Professor Lessells is Honorary Member of the ASME and holds membership in the leading professional societies both here and in England.

Daniel C. Drucker is chairman of the Division of Engineering of Brown University. He is the author of the Chapter on Three-Dimensional Photoelasticity in the "Handbook of Experimental Stress Analysis" and has published papers on oblique incidence, birefringent coatings, impact, wire rope, metal cutting, plastic stress-strain relations, limit analysis and design, uniqueness and extremum principles, soil mechanics, instability, and brittle fracture. Before coming to Brown in 1947, Professor Drucker had been Supervisor of Mechanics of Solids at Armour Research Foundation and taught at Illinois Institute of Technology and Cornell University.

Joseph Kestin is professor of engineering at Brown University. His fields of major interest are thermodynamics and fluid mechanics. One of his current studies in the accurate determination of viscosity is a part of the ASME Steam Properties Research Project which is to lead to an authoritative steam table to 15000 psi and 1500 F. Author of many research papers Professor Kestin also has translated Schmidt's "Thermodynamics," Schlichting's "Boundary Layer Theory," and Sommerfeld's "Thermodynamics." Before coming to Brown in 1952, Dr. Kestin was head of mechanical engineering at the Polish University College, London, England.

All papers to be sponsored by the ASME Applied Mechanics Division now should be sent to:

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